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SKILLS HANDBOOK LINKS
Throughout this textbook you will see links to the Skills Handbook, where appropriate, in each activity. These links identify supporting material in the Skills Handbook that will help you with that activity.
A1.1 Safety Conventions and Symbols

Although we make every effort to make the science experience a safe one, there are some inherent risks. These risks are generally associated with the materials and equipment used, and with disregard of safety instructions when conducting investigations. Most of these risks pose no more danger than we normally experience in everyday life. We can reduce these risks by doing the following: being aware of the possible hazards, knowing the rules, behaving appropriately, and using common sense.

Remember, you share the responsibility not only for your own safety, but also for the safety of those around you (Figure 1). Always alert your teacher in case of an accident. In this textbook, chemicals, equipment, and procedures that are hazardous are indicated by the appropriate Workplace Hazardous Materials Information System (WHMIS) symbol or by  .

Figure 1 Taking the proper safety precautions will ensure a safe environment for you and your classmates.

WHMIS SYMBOLS AND HHPS

The Workplace Hazardous Materials Information System (WHMIS) provides workers and students with complete and accurate information about hazardous products. All chemical products supplied to schools, businesses, and industries must contain standardized labels and be accompanied by a Material Safety Data Sheet (MSDS). The MSDS provides detailed information about the product. Clear and standardized symbols are an important component of WHMIS (Table 1, page 680). These symbols must be present on the product's original container and shown on other containers if the product is transferred.

The Canadian Hazardous Products Act requires manufacturers of consumer products containing chemicals to include a symbol that specifies the nature of the hazard and whether it is the container or the contents that is dangerous. In addition, the label must state any secondary hazards, first-aid treatment, storage, and disposal. Household Hazardous Product Symbols (HHPS) are used to show the type of hazard. The shape of the frame around the symbol indicates whether the hazard is due to the contents or the container (Figure 2).

Figure 2 Household Hazardous Product Symbols (HHPS) appear on many products. A triangular frame indicates that the container is potentially dangerous. An octagonal frame indicates that the contents pose a hazard.

A1.2 Safety in the Laboratory

Safety in the laboratory is an attitude and a habit more than it is a set of rules. It is easier to prevent accidents than to deal with the consequences of an accident. Most of the following rules are common sense:

- Do not enter a laboratory unless a teacher or other supervisor is present, or you have permission to do so.
- Know your school’s safety regulations.
- Tell your teacher about any allergies or medical problems you may have.
- Wear eye protection, a lab apron, and safety gloves when instructed by your teacher. Wear closed shoes.
- Tie back long hair and wear a protective lab coat over loose clothing. Remove any loose jewellery and finger rings.
- Keep yourself and your work area tidy and clean. Keep aisles clear.
- Never eat, drink, or chew gum in the laboratory.

(Section A1.2 continues on page 681.)
<table>
<thead>
<tr>
<th>Class and type of compounds</th>
<th>WHMIS symbol</th>
<th>Risks</th>
<th>Precautions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Class A: Compressed Gas</strong></td>
<td>![symbol]</td>
<td>• could explode due to pressure&lt;br&gt;• could explode if heated or dropped&lt;br&gt;• possible hazard from both the force of explosion and the release of contents</td>
<td>• ensure container is always secured&lt;br&gt;• store in designated areas&lt;br&gt;• do not drop or allow to fall</td>
</tr>
<tr>
<td>Material that is normally gaseous and kept in a pressurized container</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Class B: Flammable and Combustible Materials</strong></td>
<td>![symbol]</td>
<td>• may ignite spontaneously&lt;br&gt;• may release flammable products if allowed to degrade or when exposed to water</td>
<td>• store in designated areas&lt;br&gt;• work in well-ventilated areas&lt;br&gt;• avoid heating&lt;br&gt;• avoid sparks and flames&lt;br&gt;• ensure that electrical sources are safe</td>
</tr>
<tr>
<td>Materials that will continue to burn after being exposed to a flame or another ignition source</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Class C: Oxidizing Materials</strong></td>
<td>![symbol]</td>
<td>• can cause skin or eye burns&lt;br&gt;• increase fire and explosion hazards&lt;br&gt;• may cause combustibles to explode or react violently</td>
<td>• store away from combustibles&lt;br&gt;• wear body, hand, face, and eye protection&lt;br&gt;• store in container that will not rust or oxidize</td>
</tr>
<tr>
<td>Materials that can cause other materials to burn or support combustion</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Class D: Toxic Materials Immediate and Severe</strong></td>
<td>![symbol]</td>
<td>• may be fatal if ingested or inhaled&lt;br&gt;• may be absorbed through the skin&lt;br&gt;• small volumes have a toxic effect</td>
<td>• avoid breathing dust and vapours&lt;br&gt;• avoid contact with skin and eyes&lt;br&gt;• wear protective clothing, and face and eye protection&lt;br&gt;• work in well-ventilated areas and wear breathing protection</td>
</tr>
<tr>
<td>Poisons and potentially fatal materials that cause immediate and severe harm</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Class D: Toxic Materials Long-Term Concealed</strong></td>
<td>![symbol]</td>
<td>• may cause death or permanent injury&lt;br&gt;• may cause birth defects or sterility&lt;br&gt;• may cause cancer&lt;br&gt;• may be sensitzers causing allergies</td>
<td>• wear appropriate personal protection&lt;br&gt;• work in well-ventilated areas&lt;br&gt;• store in appropriate designated areas&lt;br&gt;• avoid direct contact&lt;br&gt;• use hand, body, face, and eye protection&lt;br&gt;• ensure respiratory and body protection is appropriate for the specific hazard</td>
</tr>
<tr>
<td>Materials that have a harmful effect after repeated exposures or over a long period</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Class D: Biohazardous Infectious Materials</strong></td>
<td>![symbol]</td>
<td>• may cause anaphylactic shock&lt;br&gt;• include viruses, yeasts, moulds, bacteria, and parasites that affect humans&lt;br&gt;• include fluids containing toxic products&lt;br&gt;• include cellular components</td>
<td>• special training is required to handle materials&lt;br&gt;• work in designated biological areas with appropriate engineering controls&lt;br&gt;• avoid forming aerosols&lt;br&gt;• avoid breathing vapours&lt;br&gt;• avoid contamination of people and/or area&lt;br&gt;• store in special designated areas</td>
</tr>
<tr>
<td>Infectious agents or biological toxins that cause a serious disease or death</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Class E: Corrosive Materials</strong></td>
<td>![symbol]</td>
<td>• eye and skin irritation on exposure&lt;br&gt;• severe burns/tissue damage on longer exposure&lt;br&gt;• lung damage if inhaled&lt;br&gt;• may cause blindness if they contact eyes&lt;br&gt;• environmental damage from fumes</td>
<td>• wear body, hand, face, and eye protection&lt;br&gt;• use breathing apparatus&lt;br&gt;• ensure protective equipment is appropriate&lt;br&gt;• work in well-ventilated areas&lt;br&gt;• avoid all direct body contact&lt;br&gt;• use appropriate storage containers and ensure nonventing closures</td>
</tr>
<tr>
<td>Materials that react with metals and living tissue</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Class F: Dangerously Reactive Materials</strong></td>
<td>![symbol]</td>
<td>• may react with water&lt;br&gt;• may be chemically unstable&lt;br&gt;• may explode if exposed to shock or heat&lt;br&gt;• may release toxic or flammable vapours&lt;br&gt;• may vigorously polymerize&lt;br&gt;• may burn unexpectedly</td>
<td>• handle with care avoiding vibration, shocks, and sudden temperature changes&lt;br&gt;• store in appropriate containers&lt;br&gt;• ensure storage containers are sealed&lt;br&gt;• store and work in designated areas</td>
</tr>
<tr>
<td>Materials that may have unexpected reactions</td>
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Section A1.2 continued

- Know the location of MSDS information, exits, and all safety equipment, such as the first-aid kit, fire blanket, fire extinguisher, and eyewash station, and be familiar with their contents and operation.
- Avoid moving suddenly or rapidly in the laboratory, especially near chemicals and sharp instruments.
- If you are not sure what to do, ask your teacher for directions.
- Never change anything or start an activity or investigation without your teacher's approval.
- Before you start an investigation that you have designed yourself, get your teacher's approval.
- Never attempt unauthorized experiments.
- Never work in a crowded area or alone in the laboratory.
- Always wear approved eye protection in a laboratory.
- Do not taste any substance in a laboratory.
- Do not mix harmful chemicals together in a closed area. For example, if you are using a light source in a large cardboard box, be sure you have enough holes at the top of the box and on the sides to dissipate thermal energy.
- Always wear ear protection when experimenting with loud sounds.

Handling glassware safely

- Never use glassware that is broken, cracked, or chipped. Give such glassware to your teacher or dispose of it as directed. Do not put the item back into circulation.
- Never pick up broken glassware with your fingers. Use a broom and dustpan.
- Dispose of glass fragments in special containers marked "Broken Glass."
- Check with your teacher before heating any glassware. Heat glassware only if it is approved for heating.
- Be very careful when cleaning glassware. There is an increased risk of breakage from dropping when the glassware is wet and slippery.
- If you cut yourself, inform your teacher immediately and get appropriate first aid. Embedded glass or continued bleeding requires medical attention.

Using sharp instruments safely

- Select the appropriate instrument for the task. Never use a knife when scissors would work best.
- Never change anything or start an activity or investigation without your teacher's approval.
- Never cut away from yourself and others.
- If you cut yourself, inform your teacher immediately and get appropriate first aid.

Heat safety

- Make sure that heating equipment, such as the burner, hot plate, or electric heater, is secure on the bench and clamped in place when necessary.
- Do not use a laboratory burner near wooden shelves, flammable liquids, or any other item that is combustible.
- Do not allow overheating if you are performing an experiment in a closed area. For example, if you are using a light source in a large cardboard box, be sure you have enough holes at the top of the box and on the sides to dissipate thermal energy.
- Always assume that hot plates and electric heaters are hot, and use protective gloves when handling.
- Do not touch a light source that has been on for some time. It may be hot and cause burns.
- In a laboratory where burners or hot plates are being used, never pick up a glass object without first checking the temperature by placing your hand near but not touching it. Glass and metal items that have been heated may not appear to be hot, but can cause burns.
- If you burn yourself, immediately run cold water gently over the burned area or immerse the burned area in cold water and inform your teacher.
- Never look down the barrel of a laboratory burner.
- Always pick up a burner by its base, never by its barrel.
- Never leave a lighted burner unattended.
- Any metal powder can be explosive. Do not put these in a flame.
- To heat a beaker, put it on the hot plate and secure with a ring support attached to a utility stand. (Placing a wire gauze under the beaker is optional.)
- Remember to include a cooling time in your experiment plan; do not put away hot equipment.

FIRE SAFETY
- Immediately inform your teacher of any fires. A very small fire in a container may be extinguished by covering the container with a wet paper towel or a ceramic square to cut off the supply of air. Alternatively, sand may be used to smother small fires. A bucket of sand with a scoop should be available in the laboratory.
- If anyone's clothes or hair catch fire, tell the person to drop to the floor and roll. Then use a fire blanket to smother the flames. Never wrap the blanket around a standing person on fire.
- For larger fires, immediately evacuate the area. Call the office or sound the fire alarm. Do not try to extinguish larger fires. As you leave the classroom, make sure that the windows and doors are closed.
- If you use a fire extinguisher, direct the extinguisher at the base of the fire and use a sweeping motion, moving the extinguisher nozzle back and forth across the front of the fire's base.

ELECTRICAL SAFETY
- Do not operate electrical equipment near running water or a large container of water. Water or wet hands should never be near electrical equipment such as a hot plate, a light source, or a microscope.
- Check the condition of electrical equipment. Do not use it if wires or plugs are damaged, or if the ground pin has been removed.
- If using a light source, check that the wires of the light fixture are not frayed, and that the bulb socket is in good shape and well secured to a stand.
- Make sure that electrical cords are not placed where someone could trip over them.
- When unplugging equipment, remove the plug gently from the socket. Do not pull on the cord.
- When using variable power supplies, start at low voltage and increase slowly.

WASTE DISPOSAL
Waste disposal at school, at home, and at work is a societal issue. Most laboratory waste can be washed down the drain or, if it is in solid form, placed in ordinary garbage containers. However, some waste must be treated more carefully. It is your responsibility to follow procedures and to dispose of waste in the safest possible manner according to your teacher's instructions.

FIRST AID
The following guidelines apply in case of an injury, such as a burn, cut, chemical spill, ingestion, inhalation, or splash in the eyes:
- Always inform your teacher immediately of any injury.
- If the injury is a minor cut or abrasion, wash the area thoroughly. Using a compress (for example, clean paper towels), apply pressure to the cut to stop the bleeding. When bleeding has stopped, replace the compress with a sterile bandage. If the cut is serious, apply pressure and seek medical attention immediately.
- If you get a solution in your eye, quickly use the eyewash or nearest running water. Continue to rinse the eye with water for at least 15 min. Unless you have a plumbed eyewash system, you will also need assistance in refilling the eyewash container. Have another student inform your teacher of the accident. The injured eye should be examined by a doctor.
- If the injury is a burn, immediately immerse the affected area in cold water, or run cold water gently over the burned area. This will reduce the temperature and prevent further tissue damage.
- In case of electric shock, unplug the appliance and do not touch it or the victim. Inform your teacher immediately.
- If a classmate's injury has rendered him/her unconscious, notify your teacher immediately. Your teacher will perform CPR if necessary. Do not administer CPR unless under specific instructions from your teacher. Call the school office and request emergency medical help.
A2 Scientific Inquiry

In our attempts to further our understanding of the natural world, we encounter questions, mysteries, or events that are not readily explainable. To develop explanations, we investigate using scientific inquiry. An important aspect of scientific inquiry is that science is only one of many ways people explore, explain, and come to know the world around them. Scientific inquiry is a multifaceted process that involves the following: identifying questions that can be answered through scientific investigations; using appropriate tools and techniques to gather, analyze, and interpret data; developing descriptions, explanations, predictions, and models using evidence; thinking critically and logically to make the relationships between evidence and explanations; recognizing and analyzing alternative explanations and predictions; and communicating scientific procedures and explanations.

The methods used in scientific inquiry depend, to a large degree, on the purpose of the inquiry. There are four common types of scientific inquiry: the controlled experiment, the correlational study, the observational study, and the activity. These types of scientific inquiry require specific skills. The skills are discussed below, followed by a detailed description of how they relate to each of the four types of scientific inquiry.

A2.1 Skills of Scientific Inquiry

Scientific inquiry requires certain skills that are important in the process of conducting an investigation. These skills can be organized into four categories: initiating and planning, performing and recording, analyzing and interpreting, and communicating.

INITIATING AND PLANNING

1. Questioning: Most scientific investigations begin with a question. It is important to ask the right questions. In certain types of scientific inquiry, the question must be testable. This means that it must ask about a possible cause-and-effect relationship. A cause-and-effect relationship is one in which a change in one variable (see #3) causes a change in another variable. A testable question might start in one of the following ways: What causes . . . ? How does . . . affect . . . ? If . . . , what happens to . . . ?

2. Researching: This is a skill that occurs across all four categories of scientific inquiry and includes preparing for research, accessing resources, processing information, and transferring learning. The process involves identifying the type of information that is required, using strategies to locate and access the information, recording the information, synthesizing findings, and formulating conclusions.

3. Identifying variables: Considering the variables involved in an investigation is an important step in designing an effective investigation. Variables are any factors that could affect the outcome of an investigation. There are three kinds of variables in a controlled experiment: the manipulated variable, the responding variable, and the controlled variables.

   - The manipulated variable (also known as the independent variable or cause variable) is the variable that is deliberately changed by the investigator.

   - The responding variable (also known as the dependent variable or effect variable) is the variable that the investigator believes will be affected by a change in the manipulated variable.

   - The controlled variables are variables that may affect the responding variable, but that are held constant so that they cannot affect the responding variable. A controlled experiment is a test of whether (and how) a manipulated variable affects a responding variable. To make the test fair, all other variables that may affect the responding variable are kept constant (unchanging).

4. Hypothesizing: A hypothesis is a predicted answer to the testable question. It proposes a possible explanation based on an already known scientific theory, law, or other generalization.

   A hypothesis may be written in the form of an “If . . . , then . . . because . . . ” statement. If the manipulated (independent) variable is changed in a particular way, then we predict that the responding (dependent) variable will change in a particular way, and we provide a theoretical explanation for the prediction. You may create more than one hypothesis from the same testable question. For example,

   If an air-filled balloon is placed in a freezer and its temperature is decreased, then its volume will decrease because, according to the kinetic molecular theory, atoms and molecules slow down and occupy less space at lower temperatures.

   When you conduct an investigation, your observations do not always support the prediction in your hypothesis. When this happens, you may re-evaluate and modify your hypothesis and design a new experiment.

PERFORMING AND RECORDING

1. Conducting inquiry: As you perform an investigation, follow the steps in the procedure carefully and thoroughly. Check with your teacher if you find that you need to make significant alterations to your procedure. Use all equipment and materials safely, appropriately, and with precision.

2. Making observations: When you conduct an investigation, you should make accurate observations at regular intervals and record them carefully and accurately. Record exactly what you observe. Observations from an experiment may not always be what you expect them to be. Qualitative (descriptive) and quantitative (measured) observations may be made during an investigation. Some observations may also be provided for you during an investigation.

   Quantitative observations are based on measured quantities, such as temperature, volume, and mass. They are usually recorded in data tables.

   Qualitative observations describe characteristics that cannot be expressed in numbers, such as texture, smell, and taste. They can be recorded using words, pictures, tables, or labelled diagrams.

3. Collecting, organizing, and recording data: During an investigation you should collect and record all data and observations, and organize these into formats that are easily interpreted (such as tables and charts).

ANALYZING AND INTERPRETING

1. Analyzing: Analyzing involves looking for patterns and relationships that will help you explain your results and give you new information about the question you are investigating. Your analysis will tell you whether your observations support your hypothesis.

2. Evaluating: It is very important to evaluate the evidence that is obtained through observations and analysis. When evaluating the results of an investigation, here are some aspects you should consider:

   • Experimental design: Were there any problems with the way you planned your experiment? Did you control all the variables except for the manipulated variable?
   • Equipment and materials: Was the equipment adequate? Would other equipment have been better? Was something used incorrectly? Did you have difficulty with a piece of equipment?
   • Skills: Did you have the appropriate skills for the investigation? Did you have to use a skill that you were just beginning to learn?
   • Observations: Did you accurately record all the relevant observations?

COMMUNICATING

1. It is important to share both your process and your results. Other people may want to repeat your investigation, or they may want to use or apply your results in another situation. Your write-up or report should reflect the process of scientific inquiry that you used in your investigation.

2. At this stage, you should be prepared to extend insights and opinions from your findings, suggest areas for further investigation, and relate research findings to the world around you.

In the following sections, we will detail the components of the four types of investigations: controlled experiments, correlational studies, observational studies, and activities.

A2.2 Controlled Experiments

A controlled experiment is an example of scientific inquiry in which a manipulated variable is intentionally changed to determine its effect on a responding variable. All other variables are controlled (kept constant). Controlled experiments are performed when the purpose of the inquiry is to create, test, or use a scientific concept.

The common components of controlled experiments are outlined below. Note that there are normally many cycles through the steps during an actual experiment.

TESTABLE QUESTION

A testable question forms the basis for your controlled experiment. The investigation is designed to answer the question. Controlled experiments are about relationships among variables, so your question could be about the effects on variable A when variable B is changed.

VARIABLES

The primary purpose of a controlled experiment is to determine whether a change in a manipulated variable causes a noticeable change in a responding variable while all other variables remain constant. Therefore, you must identify all major variables that you will measure and/or control in your investigation. What is the manipulated (independent) variable? What is the responding (dependent) variable? What are the controlled variables?

When conducting a controlled experiment, change only one manipulated variable at a time, holding all the others (except the responding variable) constant. This way, you can assume that the results are caused by the manipulated variable and not by any of the other variables.
HYPOTHESIS/PREDICTION
When formulating a hypothesis, first read the testable question, the experimental design, and the procedure, if provided. Then, try to identify (and distinguish) the manipulated variable, the responding variable, and the controlled variables. Your hypothesis will be a predicted answer to the testable question accompanied by a theoretical explanation for your prediction.

EXPERIMENTAL DESIGN
The design of a controlled experiment shows how you plan to answer your question. The design outlines how you will change the manipulated variable, measure any variations in the responding variable, and control all the other variables. It is a summary of your plan for the experiment.

EQUIPMENT AND MATERIALS
Make a detailed list of all equipment and materials you will use, including sizes and quantities where appropriate. Be sure to include safety equipment, such as eye protection, lab apron, protective gloves, and tongs, where needed. Draw a diagram to show any complicated setup of apparatus.

PROCEDURE
Write a step-by-step description of how you will perform your investigation. It must be clear enough for someone else to follow, and it must explain how you will deal with each of the variables in your investigation. The first step in a procedure usually refers to any safety precautions that need to be addressed, and the last step relates to any cleanup that needs to be done.

OBSERVATIONS
There are many ways you can gather and record observations during your investigation. It is helpful to plan ahead and think about what data you will need to answer the question and how best to record them (for example, data tables, pictures, or labelled diagrams may be helpful). This helps to clarify your thinking about the question posed at the beginning of the investigation, the variables, the number of trials, the procedure, the materials, and your skills. It will also help you organize your evidence for easier analysis.

ANALYZE AND EVALUATE
You will need to analyze and interpret your observations—this may include graphing your data and analyzing any patterns or trends that may be evident in your graphs. After thoroughly analyzing your observations, you may have sufficient and appropriate evidence to enable you to answer the testable question posed at the beginning of the investigation.

You must evaluate the processes that you followed to plan and perform the investigation. You will also evaluate the outcome of the investigation, which involves evaluating your hypothesis/prediction. You must identify and take into account any sources of error and uncertainty in your measurements.

Finally, compare your hypothesis/prediction with the evidence. Is your hypothesis supported by the evidence?

APPLY AND EXTEND
Reflect on how your investigation relates to the world around you: can you use what you have learned in everyday life?

REPORTING ON THE INVESTIGATION
Your lab report should describe your planning process and procedure clearly and in enough detail that the reader could repeat the experiment exactly as you performed it. You should present your observations, your analysis, and your evaluation of your experiment clearly, accurately, and honestly.

A2.3 Correlational Studies
When the purpose of scientific inquiry is to test a suspected relationship between two different variables, but a controlled experiment is not possible, a correlational study is conducted. In a correlational study, the investigator tries to determine whether one variable is affecting another without purposely changing or controlling any of the variables. Instead, variables are allowed to change naturally. It is often difficult to isolate cause and effect in correlational studies. A correlational study requires very large sample numbers and many replications to increase the certainty of the results.

A correlational study does not require experiments or fieldwork; for example, the investigator can use databases prepared by other researchers to find relationships between two or more variables. The investigator can, however, choose to make observations and measurements as well, through fieldwork, interviews, and surveys.

A hypothesis or prediction is not useful in a correlational study. Correlational studies are not intended to establish cause-and-effect relationships. However, the results of a correlational study can be used to formulate a hypothesis about the causal relationship between the variables.

The common components of a correlational study are outlined below. Even though the sequence is presented as linear, there are normally many cycles through the steps during an actual study.

PURPOSE
When planning a correlational study, it is important that you pose a question about a possible statistical relationship between variable A and variable B. Choose a topic that interests you. Determine whether you are going to replicate or revise a previous study, or create a new one. Indicate your decision in the statement of the purpose.
VARIABLES
In a correlational study you must determine whether two variables are related, without controlling any of the variables. You must identify all the major variables that will be measured and/or observed in your investigation.

STUDY DESIGN
When designing your correlational study you must identify the setting and the methods you will use in carrying out your investigation. You should describe the type(s) of data you plan to collect and its sources. Your design should address questions such as the following: Will you be conducting a survey? If so, where will the survey be conducted? Who will answer your questionnaire? When will the survey be conducted? How often will the survey be administered? If you are obtaining information from an existing database, then describe the source of the information and your plans for analyzing the information.

EQUIPMENT AND MATERIALS
Make a detailed list of all equipment and materials used, including sizes and quantities where appropriate. Be sure to include safety equipment, such as eye protection and ear protection, where needed. Draw a diagram to show any complicated setup of apparatus.

PROCEDURE
Write a step-by-step description identifying how you will gather data on the variables under study. You will also need to identify potential sources of data. There are two possible sources: observations made by you, the investigator; and existing data (databases, etc.).

OBSERVATIONS
If you are collecting your own data through observation, then you will need to plan ahead and think about what data you will need and how best to record them. There are many ways to gather and record your observations (such as data tables, pictures, or labelled diagrams). This is an important step because it helps to clarify your thinking about the question posed at the beginning, the variables, the procedure, the materials, and your skills. It will also help you organize your observations for easier analysis.

ANALYZE AND EVALUATE
You will need to analyze and interpret your observations or sourced data—this will usually include graphing the data and analyzing any patterns or trends that may be evident in your graphs. You will need to identify the relationship between your two variables. A positive correlation indicates a direct relationship between variables: an increase in one variable corresponds to an increase in another. A negative correlation indicates an inverse relationship: an increase in one variable corresponds to a decrease in the other variable. If there is no relationship between the variables, then there is no correlation.

After thoroughly analyzing your observations, you may have sufficient and appropriate evidence to enable you to answer the question you posed at the beginning of the investigation. Was there a relationship between variable A and variable B?

Evaluate the processes that you followed to plan and perform the investigation. Also evaluate the outcome of the investigation, which involves evaluating any prediction you made at the beginning of the investigation. You must identify and take into account any sources of error and uncertainty in your measurements.

APPLY AND EXTEND
Reflect on how your investigation relates to the world around you: how can you use what you have learned in your everyday life?

REPORTING ON THE INVESTIGATION
In preparing your report, your objectives should be to describe your design and procedure accurately and to report your observations, analyses, and evaluations accurately and honestly.

A2.4 Observational Studies
Often, the purpose of an inquiry is simply to study a natural phenomenon with the intention of gaining scientifically significant information to answer a question. Observational studies involve observing a subject or phenomenon in an unobtrusive or unstructured manner, usually with no specific hypothesis. The inquiry does not start off with a hypothesis, but a hypothesis may be generated as information is collected.

The stages and processes of scientific inquiry through observational studies are summarized below. Even though the sequence is presented as linear, there are normally many cycles through the steps during an actual study.

PURPOSE
When planning an observational study, it is important to pose a general question about the natural world. Choose a topic that interests you. Determine whether you are going to replicate or revise a previous study, or create a new one. Indicate your decision in a statement of the purpose.

EQUIPMENT AND MATERIALS
Make a detailed list of all equipment and materials used, including sizes and quantities where appropriate. Be sure to include safety equipment, such as eye protection, lab apron, protective gloves, and tongs, where needed. Draw a diagram to show any complicated setup of apparatus.
PROCEDURE
Write a step-by-step description of how you will make your observations. It must be clear enough for someone else to follow. The first step in a procedure usually refers to any safety precautions that need to be addressed, and the last step relates to any cleanup that needs to be done.

OBSERVATIONS
There are many ways that you can gather and record observations—including quantitative observations—during an observational study. All observations should be objective and unambiguous. Consider ways to organize your information for easier analysis.

ANALYZE AND EVALUATE
After thoroughly analyzing your observations, you may have sufficient and appropriate evidence to enable you to answer the question posed at the beginning of the investigation. You may also have enough observations and information to form a hypothesis for a controlled experiment.

At this stage of the investigation, you will evaluate the processes used to plan and perform the investigation. Evaluating the processes includes evaluating the materials, the design, the procedure, and your skills. The results of most such investigations will suggest further studies, perhaps controlled experiments or correlational studies, to explore tentative hypotheses you may have developed.

APPLY AND EXTEND
At this stage you should reflect on how your investigation relates to the world around you: how can you use what you have learned in your everyday life?

REPORTING ON THE INVESTIGATION
In your report, describe your design and procedure accurately, and report your observations accurately and honestly.

A2.5 Activities
An activity is a type of scientific inquiry that provides opportunities for you to demonstrate your knowledge and understanding of a specific concept. An activity might involve constructing a device, reasoning your way through a problem (a thought experiment), or using given information to complete a task. You might be asked to consider creative ways to present the concept, such as a performance, a digital platform, or a written report. The materials that are required for different activities can vary greatly, so materials may need special planning and attention.

Activities do not have structured stages and processes. However, to demonstrate a thorough understanding of the concept, you will need to communicate the key points. You will also need to analyze and evaluate your presentation, which may include looking at how your understanding of the concept has grown and applying what you have learned to other areas.

A2.6 Lab Reports
When carrying out investigations, it is important that scientists keep records of their plans and results, and share their findings. In order to have their investigations repeated (replicated) and accepted by the scientific community, scientists generally share their work by publishing reports in which details of their design, materials, procedure, evidence, analysis, and evaluation are provided.

Lab reports are prepared after an investigation is completed. To ensure that you can accurately describe the investigation, keep thorough and accurate records of your activities as you carry out the investigation. Your lab book or report should reflect the type of scientific inquiry that you used in the investigation (controlled experiment, correlational study, observational study, or activity) and should be based on the following headings, as appropriate:

TITLE
At the beginning of your report, write the number and title of your investigation. In this book, the title is always given, but if you are designing your own investigation, create a title that suggests what the investigation is about. Include the date the investigation was conducted and the names of all lab partners (if you worked as a team).

PURPOSE
State the purpose of the investigation. Why are you doing this investigation?

TESTABLE QUESTION
State the question that you attempted to answer in the investigation. Sometimes the question is provided for you; other times you are expected to formulate your own. If it is appropriate to do so, state the question in terms of manipulated and responding variables.

HYPOTHESIS/PREDICTION
For a controlled experiment you will usually have to compose a hypothesis or prediction. This will be a proposed answer to your testable question. When writing a hypothesis include both a prediction and a reason for the prediction, based on scientific theory, law, or other generalization. You may use the “If . . . , then . . . because . . . ” form. A simple prediction may be written in the “If . . . , then . . . ” form.

VARIABLES
Identify all major variables that you measured and/or controlled in the investigation. What is the manipulated variable? What is the responding variable? What are the major controlled variables?
EXPERIMENTAL DESIGN
Provide a brief general overview (one to three sentences) of what you did in your investigation. If your investigation involved manipulated, responding, and controlled variables, list them and indicate how they were changed, measured, or held constant. Identify any control or control group that was used in the investigation.

EQUIPMENT AND MATERIALS
Include a detailed list of all the equipment and materials that you used, including sizes and quantities where appropriate. Be sure to include safety equipment, such as eye protection, lab apron, protective gloves, and tongs, where needed. Draw a diagram to show any complicated setup of apparatus.

PROCEDURE
Describe, in detailed, numbered steps, the procedure you followed to carry out your investigation. Your teacher may specify which style you should use. Examples of three common writing styles are
1. third person past tense (“The test tubes were heated . . .”)
2. first person plural past tense (“We heated the test tubes . . .”)
3. second person imperative (“Heat the test tubes . . .”)
Include steps to clean up and dispose of waste and all safety considerations.

OBSERVATIONS
Present your observations in a form that is easily understood. This includes all the qualitative and quantitative observations that you made. Be as precise as possible when describing quantitative observations. Include any unexpected observations and present your information clearly. If you have only a few observations, this could be a list; for controlled experiments and for many observations, a data table, labelled diagram, or written description would be more appropriate.

ANALYSIS
Complete the questions found in the Analyze and Evaluate section of the investigation. These questions will prompt you to analyze and interpret your observations, answer a testable question, draw conclusions, and evaluate both your experiment and your conclusions. You will also be prompted to graph your data and analyze these graphs where applicable.

If you are writing up an investigation for which there are no questions, write your own analysis. Interpret your observations and present the evidence in the form of titled tables, graphs, or illustrations, as appropriate. Include any calculations, the results of which can be shown in a table. Make statements about any patterns or trends you observed. Conclude the analysis with a statement based only on the evidence you have gathered, answering the question that initiated the investigation.

EVALUATION
The evaluation is your judgment about the quality of evidence obtained and about the validity of the prediction and hypothesis (if present). This section can be divided into two parts:
1. Evaluation of the experiment: Did your experiment provide reliable and valid evidence to enable you to answer the question? Consider the experimental design, the procedure, and your laboratory skills. Were they all adequate? Are you confident enough in the evidence to use it to evaluate any prediction and/or hypothesis you made?
2. Evaluation of the prediction: Was the prediction you made before the investigation supported or falsified by the evidence? Based on your evaluation of the evidence and prediction, is the hypothesis you used to make your prediction supported, or should it be rejected?

APPLY AND EXTEND
Answer any Apply and Extend questions in the investigation. Number your answers as they appear in the Apply and Extend section in the textbook.
Communicating in Science

Advances in science and our understanding of the natural world are the result of scientists sharing ideas and information. Watson and Crick won the Nobel Prize for discovering the double helix structure of DNA, but they solved this structure with the help of X-ray photographs taken by Rosalind Franklin during her own research.

It is important for scientists to share ideas and research with other scientists. New research findings are shared at conferences and in journal publications. Research scientists take pride in being published. Being published confirms that the research adds to the knowledge base of the scientific community. Sharing information helps to spread knowledge, solve problems, and inspire other scientists.

The Scientific Journal

Scientific journals are publications that are used to present new research. There are thousands of different science journals published worldwide, and in many languages. A scientific journal may be specific to a subject (for example, Canadian Journal of Physics) or contain articles that cover a variety of subjects within a field (for example, Nature). These publications may be electronic (online) or in print and may be published weekly, monthly, bimonthly, or quarterly.

Peer Review

When an article is submitted to a journal, the research findings are critically reviewed by experts in the topic. This ensures that the research presents ideas that are supported by practices of good science. High-quality evidence and appropriate conclusions are necessary for the article to be accepted for publication. An article that is submitted to a reputable journal may take months to be approved for publication. The article may be returned to the author(s) for revision if necessary.

Scientists aim to be published in the most respected journals. The peer-review process contributes to the reputation of a journal. It helps to maintain standards and provide credibility. A scientific journal becomes reputable by ensuring that only articles of high quality are published. Very prestigious journals (such as Nature, Science, and Physics Review Letters) are known for publishing research backed by only the best practices of science. Reputable journals are widely read and considered reliable by the scientific community. Publication in a reputable journal brings immediate recognition for the author(s). Research that is not published in a peer-reviewed journal is often overlooked.

Format of Research Articles

Research articles have specific sections. Articles often include an abstract, introduction, methods, results, discussion, conclusions, and references. Go to the Nelson Science website to see an example of a real research article with a description of each section.

THE ABSTRACT

The abstract is a short summary of the article. It presents the purpose of the research, outlines the design of the experiment and methods used, and summarizes findings or conclusions. A well-written abstract is useful when looking for articles with specific information. Time may be saved by reading the abstract and then deciding if the article is going to be helpful in supporting research. The background material, methods used, results, main subject, and discussion are all summarized within the abstract.

CITING SOURCES AND GIVING CREDIT

Once a scientist has found useful sources and included them in an article or paper, information must be provided about the article so that someone else who is interested in learning more will be able to find it. This also shows the reader that information supporting the research is current. More importantly, citing another scientist’s work provides a measure of value of what he or she has published. It gives credit to the scientist. More citations often mean that the work is worthwhile and influential. An example of this is the frequency of citation of Kary Mullis’s work that presented the polymerase chain reaction (PCR). Thousands of citations occurred within a few years of publication, confirming that his work was influential.

Further Reading

A4 Exploring Issues and Applications

Throughout this textbook you will have many opportunities to examine the connections between science, technology, society, and the environment (STSE) by exploring issues and applications.

An issue is a situation in which several points of view need to be considered in order to make a decision. There can be many positions on an issue, generally determined by the values that an individual or a society holds. Which solution is "best" is a matter of opinion; ideally, the solution that is implemented is the one that is most appropriate for society as a whole. Researching information about an issue will help you make an educated decision about it. All the skills listed in Section A4.1 may be useful in an activity that involves exploring an issue.

Scientific research produces knowledge or understanding of natural phenomena. Technologists and engineers look for ways to apply this knowledge in the development of practical products and processes. Technological inventions and innovations can have wide-ranging applications for, and impacts on, society and the environment. The purpose of exploring an application is to research a particular technological invention or innovation to determine how it works, how it is used, and how it may affect society and the environment. The skills of researching, communicating, and evaluating may be useful in an activity that involves exploring an application.

A4.1 Research Skills

The following skills are involved in many types of research. Some of these skills will help you research issues only, while some will help you research issues and applications. Refer to this section when you have questions about any of the following skills and processes.

DEFINING THE ISSUE

When exploring an issue, the first step in understanding the issue is to explain why it exists, the problems associated with it, and, if applicable, the individuals or groups, also known as stakeholders, that are involved in it. The issue includes information about the role a person takes when thinking about an issue as well as a description of who your audience will be. You could brainstorm questions involving Who? What? Where? When? Why? and How? Develop background information on the issue by clarifying facts and concepts, and identifying relevant attributes, features, or characteristics of the problem.

RESEARCHING

When beginning your research for both issues and applications, you need to formulate a research question that helps to limit, narrow, or define the scope of your research. You then need to develop a plan to find reliable and relevant sources of information. This includes outlining the stages of your research: gathering, sorting, evaluating, selecting, and integrating relevant information. You should gather information from a variety of sources if possible (for example, print, web, and personal interviews).

As you collect information, do your best to ensure that the information is reliable, accurate, and current. Avoid biased opinions, opinions that are not supported by, or that ignore, credible evidence. It is important to ensure that the information you have gathered addresses all aspects of the issue or application you are researching.

IDENTIFYING ALTERNATIVES

When exploring an issue, examine the situation and think of as many alternative solutions as you can. Be creative about combining the solutions. At this point, it does not matter whether or not the solutions seem unrealistic. To analyze the alternatives, you should examine the issue from a variety of perspectives. Stakeholders may bring different viewpoints to an issue, which may influence their position on the issue. Brainstorm or hypothesize how different stakeholders would feel about your alternatives.

ANALYZING THE ISSUE

An important part of exploring an issue is analyzing the issue. First, you should establish criteria for evaluating your information to determine its relevance and significance. You can then evaluate your sources, determine what assumptions may have been made, and assess whether you have enough information to make your decision.

To analyze an issue effectively,

- establish criteria for determining the relevance and significance of the data you have gathered
- evaluate the sources of information
- identify and determine what assumptions have been made
- challenge unsupported evidence
- evaluate the alternative solutions, possibly by conducting a risk–benefit analysis

Once the issue has been analyzed, you can begin to evaluate the alternative solutions. You may decide to carry out a risk–benefit analysis—a tool that enables you to look at each possible result of a proposed action and helps you make a decision. (See Section A4.2 for more information on risk–benefit analysis.)
DEFENDING A DECISION
After analyzing your information on your issue, you can answer your research question and take an informed position or draw a conclusion on the issue. If you are working as a group, this is the stage where everyone gets a chance to share ideas and information gathered about the issue. Then the group needs to evaluate all the possible alternatives and decide on their preferred solution based on the criteria.

Your position on the issue or conclusion must be justified using supporting information that you have researched. You should be able to defend your position to people with different perspectives. Ask yourself the following questions:

- Do I have supporting evidence from a variety of sources?
- Can I state my position clearly?
- Can I show why this issue is relevant and important to society?
- Do I have solid arguments (with solid evidence) supporting my position?
- Have I considered arguments against my position, and identified their faults?
- Have I analyzed the strong and weak points of each perspective?

COMMUNICATING
When exploring an issue, there are several things to consider when communicating your decision. You need to state your position clearly and take into consideration who your audience is. You should always support your decision with objective data and a persuasive argument if possible. Be prepared to defend your position against any opposition.

You should be able to defend your solution in an appropriate format—debate, class discussion, speech, position paper, multimedia presentation, brochure, poster, video, etc.

When exploring an application you should communicate the “need or want” for the application (why the application was developed in the first place), the “how” (how the application/technology actually works), and the risks and benefits to society, individuals, and the environment. You should conclude with your “assessment” of the application.

EVALUATING
The final phase of your decision making when exploring an issue includes evaluating the decision itself and the process used to reach the decision. After you have made a decision, carefully examine the thinking that led to your decision. Some questions to guide your evaluation include:

- What was my initial perspective on the issue? How has my perspective changed since I first began to explore the issue?
- How did we make our decision? What process did we use? What steps did we follow?
- To what extent were my arguments factually accurate and persuasively made?
- In what ways does our decision resolve the issue?
- What are the likely short- and long-term effects of the decision?
- To what extent am I satisfied with the final decision?
- What reasons would I give to explain our decision?
- If we had to make this decision again, what would I do differently?

A4.2 Risk–Benefit Analysis Model
Risk-benefit analysis is a tool used to organize and analyze information gathered in research, especially when exploring a socio-scientific issue. A thorough analysis of the risks and benefits associated with each alternative solution can help you decide on the best alternative.

- Research as many aspects of the situation as possible. Look at it from different perspectives.
- Collect as much evidence as you can, including reasonable projections of likely outcomes if the proposal is adopted.
- Classify every individual potential result as being either a benefit or a risk.
- Quantify the size of the potential benefit or risk (perhaps as a dollar figure, or a number of lives affected, or on a scale of 1 to 5).
- Estimate the probability (percentage) of that event occurring.
- By multiplying the size of a benefit (or risk) by the probability of its happening, you can calculate a probability value for each potential result.
- Total the probability values of all the potential risks, and all the potential benefits.
- Compare the sums to help you decide whether to accept the proposed action.
A5 Math Skills

A5.1 Scientific Notation

It is difficult to work with very large or very small numbers when they are written in common decimal notation. Usually it is possible to accommodate such numbers by changing the SI prefix so that the number falls between 0.1 and 1000. For example, 237 000 000 mm can be expressed as 237 km, and 0.000 000 895 kg can be expressed as 0.895 mg. However, this prefix change is not always possible, either because an appropriate prefix does not exist or because it is essential to use a particular unit of measurement in a calculation. In these cases, the best method of dealing with very large and very small numbers is to write them using scientific notation. Scientific notation expresses a number by writing it in the form $a \times 10^n$, where $1 < |a| < 10$ and the digits in the coefficient $a$ are all significant. Table 1 shows situations where scientific notation would be used.

Table 1 Examples of Scientific Notation

<table>
<thead>
<tr>
<th>Expression</th>
<th>Common decimal notation</th>
<th>Scientific notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>“124.5 million kilometres”</td>
<td>124 500 000 km</td>
<td>$1.245 \times 10^8$ km</td>
</tr>
<tr>
<td>“154 thousand picometres”</td>
<td>154 000 pm</td>
<td>$1.54 \times 10^5$ pm</td>
</tr>
<tr>
<td>“602 sextillion molecules”</td>
<td>602 000 000 000 000 000 molecules</td>
<td>$6.02 \times 10^{23}$ molecules</td>
</tr>
</tbody>
</table>

To multiply numbers in scientific notation, multiply the coefficients and add the exponents. To divide numbers in scientific notation, divide the coefficients and subtract the exponents. The answer is always expressed in scientific notation. Note that the coefficient should always be between 1 and 10. For example,

$$(4.73 \times 10^5 \text{ m})(5.82 \times 10^7 \text{ m}) = 27.5 \times 10^{12} \text{ m}^2 = 2.75 \times 10^{13} \text{ m}^2$$

$$\frac{6.4 \times 10^6 \text{ m}}{2.2 \times 10^5 \text{ s}} = 2.9 \times 10^3 \text{ m/s}$$

When evaluating exponents, the following rules apply:

- $x^a \cdot x^b = x^{a+b}$
- $(xy)^b = x^b y^b$
- $\frac{x^a}{x^b} = x^{a-b}$
- $(\frac{x}{y})^b = \frac{x^b}{y^b}$
- $(x^a)^b = x^{ab}$

SCIENTIFIC NOTATION WITH CALCULATORS

On many calculators, scientific notation is entered using a special key, labelled EXP or EE. This key includes “$\times 10^n$” from the scientific notation; you need to enter only the exponent. For example, to enter

- $7.5 \times 10^4$ press 7.5 EXP 4
- $3.6 \times 10^{-3}$ press 3.6 EXP +/- 3

Depending on the type of calculator you have, +/- may need to be entered after the relevant number.

A5.2 Uncertainty in Measurements

There are two types of quantities that are used in science: exact values and measurements. Exact values include defined quantities (1 m = 100 cm) and counted values (5 beakers or 10 trials). Measurements, however, are not exact because there is some uncertainty or error associated with every measurement.

SIGNIFICANT DIGITS

The certainty of any measurement is communicated by the number of significant digits in the measurement. In a measured or calculated value, significant digits are the digits that are known reliably, or for certain, and include the last digit that is estimated or uncertain. Significant digits include all digits correctly reported from a measurement.

Follow these rules to decide if a digit is significant:

1. All non-zero digits are significant.
2. If a decimal point is present, zeros to the left of other digits (leading zeros) are not significant.
3. If a decimal point is not present, zeros to the right of the last non-zero digit (trailing zeros) are not significant.
4. Zeros placed between other digits are always significant.
5. Zeros placed after other digits to the right of a decimal point are significant.
6. When a measurement is written in scientific notation, all digits in the coefficient are significant.
7. Counted and defined values have infinite significant digits.

Table 2, on the next page, shows examples of significant digits.
Table 2  Certainty in Significant Digits

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Number of significant digits</th>
</tr>
</thead>
<tbody>
<tr>
<td>32.07 m</td>
<td>4</td>
</tr>
<tr>
<td>0.0041 g</td>
<td>2</td>
</tr>
<tr>
<td>$5 \times 10^5$ kg</td>
<td>1</td>
</tr>
<tr>
<td>7002 N-m</td>
<td>4</td>
</tr>
<tr>
<td>6400 s</td>
<td>2</td>
</tr>
<tr>
<td>6.0000 A</td>
<td>5</td>
</tr>
<tr>
<td>204.0 cm</td>
<td>4</td>
</tr>
<tr>
<td>10.0 kJ</td>
<td>3</td>
</tr>
<tr>
<td>100 people (counted)</td>
<td>infinite</td>
</tr>
</tbody>
</table>

An answer obtained by multiplying and/or dividing measurements is rounded to the same number of significant digits as the measurement with the fewest significant digits, for example, using a calculator to solve the following equation:

$$77.8 \text{ km/h} \times 0.8967 \text{ h} = 69.763 \text{ km}$$

However, the certainty of the answer is limited to three significant digits, so the answer is rounded up to 69.8 km. The same applies to scientific notation. For example,

$$(5.5 \times 10^3) + (4.236 \times 10^3) = 9.7 \times 10^3$$

ROUNDING

When adding or subtracting measurements of different precisions, the answer is rounded to the same precision as the least precise measurement. For example, using a calculator,

$$11.7 \text{ cm} + 3.29 \text{ cm} + 0.542 \text{ cm} = 15.532 \text{ cm}$$

The answer must be rounded to 15.5 cm because the first measurement limits the precision to a tenth of a centimetre. Follow these rules to round answers to calculations:

1. When the first digit to be dropped is 4 or less, the last digit retained should not be changed.
   3.141 326 rounded to 4 digits is 3.141

2. When the first digit to be dropped is greater than 5, or if it is a 5 followed by at least one digit other than zero, the last digit retained is increased by 1 unit.
   2.221 682 rounded to 5 digits is 2.2217
   4.168 501 rounded to 4 digits is 4.169

3. When the first digit discarded is 5 followed by only zeros, the last digit retained is increased by 1 if it is odd, but not changed if it is even.
   2.35 rounded to 2 digits is 2.4
   2.45 rounded to 2 digits is 2.4
   $-6.35$ rounded to 2 digits is $-6.4$

MEASUREMENT ERROR

There are two types of measurement errors: random error and systematic error. Random error results when an estimate is made to obtain the last significant digit for any measurement. The size of the random error is determined by the precision of the measuring instrument. For example, when measuring length with a measuring tape, it is necessary to estimate between the marks on the measuring tape. If these marks are 1 cm apart, the random error will be greater and the precision will be less than if the marks are 1 mm apart. Such errors can be reduced by taking the average of several readings.

Systematic error is associated with an inherent problem with the measuring system, such as the presence of an interfering substance, incorrect calibration, or room conditions. For example, if a balance is not zeroed at the beginning, all measurements will have a systematic error; using a slightly worn metre stick will also introduce a systematic error. Such errors are reduced by adding or subtracting the known error, calibrating the instrument, or performing a more complex investigation.

PRECISION AND ACCURACY

Precision and accuracy are two other terms used to describe how close a measurement is to a true value. The precision of a measurement depends upon the gradations of the measuring device. Precision is the place value of the last measurable digit. For example, a measurement of 12.74 cm is more precise than a measurement of 127.4 cm because the first value was measured to hundredths of a centimetre, whereas the latter was measured to tenths of a centimetre.

No matter how precise a measurement is, it still may not be accurate. Accuracy refers to how close a value is to its accepted value. An accurate measurement has a low uncertainty. Figure 1 shows an analogy between precision and accuracy; the positions of golf balls on a golf course.

![Figure 1](image_url)

In (a) the results are precise and accurate, in (b) they are precise but not accurate, and in (c) they are neither precise nor accurate.
How certain you are about a measurement depends on two factors: the precision of the instrument used and the size of the measured quantity. More precise instruments give more certain values. For example, a mass measurement of 13 g is less precise than a measurement of 12.76 g; you are more certain about the second measurement than the first. Certainty also depends on the size of the measurement. For example, consider the measurements 0.4 cm and 15.9 cm; both have the same precision. However, if the measuring instrument is precise to ± 0.1 cm, the first measurement is 0.4 ± 0.1 cm (0.3 cm or 0.5 cm) for an error of 25%, whereas the second measurement could be 15.9 ± 0.1 cm (15.8 cm or 16.0 cm) for an error of 0.6%. For both factors—the precision of the instrument used and the value of the measured quantity—the more digits there are in a measurement, the more certain you are about the measurement.

REPORTING DATA INVOLVING MEASUREMENTS

A formal report of an experiment involving measurements should include an analysis of uncertainty, percentage uncertainty, and percentage error or percentage difference. Uncertainty is often assumed to be plus or minus half of the smallest division of the scale on the instrument; for example, the estimated uncertainty of 15.8 cm is ± 0.05 cm or ± 0.5 mm. Whenever calculations involving addition or subtraction are performed, the uncertainties accumulate. Thus, to determine the total uncertainty, the individual uncertainties must be added. For example,

\[(34.7 \text{ cm} ± 0.05 \text{ cm}) - (18.4 \text{ cm} ± 0.05 \text{ cm}) = 16.3 \text{ cm} ± 0.10 \text{ cm}\]

Percentage uncertainty is calculated by dividing the uncertainty by the measured quantity and multiplying by 100. Use your calculator to prove that 28.0 cm ± 0.05 cm has a percentage uncertainty of ± 0.18%.

Whenever calculations involving multiplication or division are performed, the percentage uncertainties must be added. If desired, the total percentage uncertainty can be converted back to uncertainty. For example, consider the area of a certain rectangle:

\[A = lw\]
\[= (28.0 \text{ cm} ± 0.18 \%)(21.5 \text{ cm} ± 0.23 \%)\]
\[= 602 \text{ cm}^2 ± 0.41 \%\]
\[A = 602 \text{ cm}^2 ± 2.5 \text{ cm}^2\]

Percentage error can be determined only if it is possible to compare a measured value with the most commonly accepted value. The equation is

\[% \text{ error} = \frac{\text{measured value} - \text{accepted value}}{\text{accepted value}} \times 100\]

Percentage difference is useful for comparing two measurements when the true measurement is not known or for comparing a measured value to a predicted value. The percentage difference is calculated as

\[% \text{ difference} = \frac{\text{measured value} - \text{predicted value}}{\text{predicted value}} \times 100\]

A5.3 Use of Units

When solving problems in science, it is important to denote the units that go with a numerical value. The number 170 as an answer is unacceptable since there are no units. The quantity 170 g/mL denotes a density, 170 °C denotes a temperature, 170 K denotes a temperature in kelvins, and 170 kPa denotes a pressure. Understanding and placing units with a value gives the proper context of the value.

You can also identify a formula by looking at the units. For instance, if a density of 9.01 g/cm³ is given, you can note that the density units have grams (mass) divided by cm³ (volume), so the formula for density is mass divided by volume or \[D = \frac{M}{V} \].

A5.4 Mathematical Equations

Several mathematical equations involving geometry, algebra, and trigonometry can be applied in physics.

ABSOLUTE VALUE

Mathematicians tell us that the absolute value is the magnitude of a real number without regard to its sign. This means that, mathematically, if \[a = 3\] and \[b = -3\], both \(a\) and \(b\) have an absolute value of 3. This can be expressed as

\[|a| = |b| = 3\]

Notice the “absolute value bars” that are shown around the letters \(a\) and \(b\). These bars indicate that we are taking the absolute value of each of these quantities.

“Magnitude” is a term frequently used by physicists. The magnitude of a quantity is the same as its absolute value. Magnitude is often referred to in problems involving vectors. In the previous example, we could have stated that \(a\) and \(b\) both have a magnitude of 3.

Linear vectors: Vectors have both magnitude and direction. Two displacement vectors are represented below:

\[\Delta \vec{d}_1 = 10 \text{ m} [\text{E}]; \Delta \vec{d}_2 = 10 \text{ m} [\text{W}]\]

While these two vectors point in opposite directions, they have a common magnitude of 10 m. Algebraically, we can state that

\[|\Delta \vec{d}_1| = |\Delta \vec{d}_2| = 10 \text{ m}\]

We can state this in words as “the magnitude of displacement one equals the magnitude of displacement two, which equals 10 m.” Note that the magnitude of a vector does not include the use of a sign or direction.
Absolute value bars can also be used in vector equations where only the magnitude of a calculated vector is required. For example, we can write Coulomb's law, which describes the force between two charged particles, as

\[ \vec{F} = k \frac{|q_1| |q_2|}{r^2} \]

This equation only indicates the magnitude of the electric force, not the direction associated with it.

Perpendicular vectors: Figure 2 shows two perpendicular displacement vectors joined tip to tail. A common method for adding two vectors algebraically involves using the Pythagorean theorem to determine the magnitude of the resultant vector.

\[ \Delta \vec{d}_1 = 18 \text{ m [E 34° N]} \]
\[ \Delta \vec{d}_2 = 10 \text{ m [N]} \]
\[ \Delta \vec{d}_3 = 15 \text{ m [E]} \]

Figure 2 Use the Pythagorean theorem to determine the resultant vector when two perpendicular displacement vectors are added tip to tail.

The Pythagorean theorem is a scalar equation. We may use this equation for vector problems so long as we only use the magnitude of the given vectors in our calculation. For this example, the Pythagorean theorem can be expressed as

\[ |\Delta \vec{d}_1|^2 = |\Delta \vec{d}_2|^2 + |\Delta \vec{d}_3|^2 \]

Note that it would be algebraically incorrect to write this equation as

\[ \Delta \vec{d}_1^2 = \Delta \vec{d}_2^2 + \Delta \vec{d}_3^2 \]

This equation is meaningless because it requires us to square vectors. In this example, \( \Delta \vec{d}_1^2 \) could have an associated unit of \((\text{m [E]})^2\), which has no physical meaning.

As a result, we must use only the magnitude of each vector if our use of the Pythagorean theorem is to be valid.

Centripetal force: Figure 3 shows an object moving with uniform circular motion as it moves counterclockwise in a circle. The velocity and acceleration are shown at two points in the object’s path. Since the object is moving with uniform circular motion, the magnitude of the velocity at each point is constant. From the diagram, it is clear that the direction of the velocity vector is different at the two points shown. Similarly, the centripetal acceleration is constant in magnitude at both points; however, its direction is different at each location. Centripetal acceleration always points toward the centre of the circle.

\[ \vec{F}_c = \frac{m v^2}{r} \]

Notice that absolute value bars have been placed around the centripetal force vector and the velocity vector. This is again because the velocity vector is squared, and squaring a vector has no physical meaning. Since the magnitude of the centripetal force and the magnitude of the object’s velocity are constant throughout the object’s motion, this use of absolute value bars is valid. Often this equation is simplified as \( F_c = \frac{m v^2}{r} \), where \( F_c \) denotes the magnitude of the centripetal force vector and \( v \) represents the magnitude of the velocity vector, or speed.

**GEOMETRY**

For a rectangle of length \( l \) and width \( w \), the perimeter \( P \) and the area \( A \) are

\[ P = 2l + 2w \quad \text{and} \quad A = lw \]

For a triangle of base \( b \) and altitude \( h \), the area is

\[ A = \frac{1}{2} bh \]

For a circle of radius \( r \), the circumference \( C \) and the area \( A \) are

\[ C = 2\pi r \quad \text{and} \quad A = \pi r^2 \]

For a sphere of radius \( r \), the area \( A \) and volume \( V \) are

\[ A = 4\pi r^2 \quad \text{and} \quad V = \frac{4}{3} \pi r^3 \]

For a right circular cylinder of height \( h \) and radius \( r \), the area and volume are

\[ A = 2\pi r^2 + 2\pi rh \quad \text{and} \quad V = \pi r^2 h \]
**ALGEBRA**

**Quadratic formula:** Given a quadratic equation in the form 
\[ ax^2 + bx + c = 0, \]
\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

In this equation, the discriminant \( b^2 - 4ac \) indicates the number of real roots of the equation. When \( b^2 - 4ac < 0 \), the quadratic function has no real roots. When \( b^2 - 4ac = 0 \), the quadratic function has one real root. When \( b^2 - 4ac > 0 \), the quadratic function has two real roots.

---

**Example 1**

A ball is launched from the roof of a building that is 40.0 m tall. The ball is launched at an angle such that its vertical velocity is 10.0 m/s and its horizontal velocity is 17.3 m/s. Determine the time the ball takes to reach the ground.

**Given:** \( \Delta h_y = -40.0 \) m; \( v_y = 10.0 \) m/s; \( \bar{v}_y = -9.8 \) m/s²

Notice that in the given information, the vertical displacement and acceleration are listed as negative values. This indicates that both of these values are vectors that are pointing downward.

We can describe the vertical motion of the ball using the equation
\[ \Delta h_y = \bar{v}_y \Delta t + \frac{1}{2} \bar{g}_y \Delta t^2 \]

**Required:** \( \Delta t \)

**Analysis:**
\[ \Delta h_y = \bar{v}_y \Delta t + \frac{1}{2} \bar{g}_y \Delta t^2 \]

**Solution:**
\[ \Delta h_y = \bar{v}_y \Delta t + \frac{1}{2} \bar{g}_y \Delta t^2 \]
\[ -40 \text{ m} = (10.0 \text{ m/s}) \Delta t - \frac{1}{2} (9.8 \text{ m/s}^2) \Delta t^2 \]
\[ 0 = -4.9 \Delta t^2 + 10.0 \Delta t + 40.0 \]

This is a quadratic equation for the variable \( \Delta t \). We can now substitute values into the quadratic formula to determine the correct values for time.

\[ a = -4.9, \ b = 10.0, \text{ and } c = 40.0 \]
\[ \Delta t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]
\[ \Delta t = \frac{-10.0 \pm \sqrt{(10.0)^2 - 4(-4.9)(40.0)}}{2(-4.9)} \]
\[ \Delta t = \frac{-10.0 \pm 29.7}{-9.8} \]
\[ \Delta t = -2.0\text{ s or } \Delta t = 4.1\text{ s} \] (Choose the positive root.)

**Statement:** It takes the ball 4.1 s to reach the ground.

---

**TRIGONOMETRY**

The first application of trigonometry was to solve right-angled triangles. Trigonometry derives from the fact that for similar triangles, the ratio of corresponding sides will be equal.

The right-angled triangle in Figure 4 shows the three sides of the triangle labelled with reference to the indicated angle \( \alpha \) (alpha). Notice that the opposite side is opposite \( \alpha \), the hypotenuse side is the longest side of the triangle, and the adjacent side is beside the angle \( \alpha \). Trigonometry provides us with three ratios that can be used for solving problems:

\[ \sin \alpha = \frac{\text{opp}}{\text{hyp}} \]
\[ \cos \alpha = \frac{\text{adj}}{\text{hyp}} \]
\[ \tan \alpha = \frac{\text{opp}}{\text{adj}} \]

**Figure 4**

Figure 5 shows a right-angled triangle where one interior angle and the length of one side are given. We can use the trigonometric ratios shown above to determine the length of the two unknown sides.

\[ \sin 54^\circ = \frac{Y}{\text{hyp}} \]
\[ \cos 54^\circ = \frac{X}{\text{hyp}} \]
\[ \sin 54^\circ = \frac{Y}{2.3\text{ cm}} \]
\[ \cos 54^\circ = \frac{X}{2.3\text{ cm}} \]
\[ Y = (2.3\text{ cm}) \sin 54^\circ \]
\[ X = (2.3\text{ cm}) \cos 54^\circ \]

\[ Y = 1.9\text{ cm} \]
\[ X = 1.4\text{ cm} \]

**Figure 5**

**Pythagorean theorem:** For the right-angled triangle in Figure 6, \( c^2 = a^2 + b^2 \), where \( c \) is the hypotenuse and \( a \) and \( b \) are the other sides.

**Figure 6**

With a right-angled triangle, use the Pythagorean theorem.
Figure 7 shows a right-angled triangle with an unknown interior angle $\beta$ (beta). We can use one of the trigonometric ratios to solve for this interior angle, since we are given the lengths of two sides of this triangle.

$$\tan \beta = \frac{\text{opp}}{\text{adj}}$$

$$\tan \beta = \frac{X}{Y}$$

$$\beta = \tan^{-1} \left( \frac{2.6 \text{ cm}}{7.3 \text{ cm}} \right)$$

$$\beta = 20^\circ$$

The length of the third side, $Z$, can be determined by using trigonometry, or by using the Pythagorean theorem. The Pythagorean theorem gives

$$Z^2 = X^2 + Y^2$$

$$Z = \sqrt{X^2 + Y^2}$$

$$Z = \sqrt{(2.6 \text{ cm})^2 + (7.3 \text{ cm})^2}$$

$$Z = 7.7 \text{ cm}$$

For the obtuse triangle in Figure 8 with angles $A$, $B$, and $C$, and opposite sides $a$, $b$, and $c$:

**Sum of the angles:** $A + B + C = 180^\circ$

**Sine law:**

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

To use the sine law, two sides and an opposite angle (SSA) or two angles and one side (AAS) must be known.

The sine law can yield an acute angle rather than the correct obtuse angle when solving for an angle greater than 90°. This problem occurs because for an angle $A$ between 0° and 90°, $\sin A = \sin (A + 90^\circ)$. To avoid this problem, always check the validity of the angle opposite the largest side of a triangle.

**Cosine law:**

$$c^2 = a^2 + b^2 - 2ab \cos C$$

To use the cosine law, three sides (SSS), or two sides and the contained angle (SAS) must be known. Notice in the cosine law that if $C = 90^\circ$, the equation reduces to the Pythagorean theorem.

**Trigonometric identities:** You may find the following trigonometric identities useful:

$$\cos \theta = \sin(90^\circ - \theta)$$

$$\sin \theta = \cos(90^\circ - \theta)$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\sin 2\theta = 2\sin \theta \cos \theta$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

### A5.5 Analyzing Experimental Data

Controlled physics experiments are conducted to determine the relationship between variables. The experimental data can be analyzed in a variety of ways to determine how the dependent variable depends on the independent variable(s). Often the resulting derived relationship can be expressed as an equation.

**PROPORTIONALITY STATEMENTS AND GRAPHING**

The statement of how one quantity varies in relation to another is a proportionality statement, or a variation statement. Common proportionality statements are as follows:

$$y \propto x$$  (direct proportion)

$$y \propto \frac{1}{x}$$  (inverse proportion)

$$y \propto x^2$$  (square proportion)

$$y \propto \frac{1}{x^2}$$  (inverse square proportion)

A proportionality statement can be converted into an equation by replacing the proportionality sign with an equal sign and including a proportionality constant. Using $k$ to represent this constant, the proportionality statements become the following equations:

$$y = kx$$

$$y = \frac{k}{x}$$

$$y = kx^2$$

$$y = \frac{k}{x^2}$$
The constant of proportionality can be determined by using graphing software or by applying regular graphing techniques as outlined in the following steps:

1. Plot a graph of the dependent variable as a function of the independent variable. If the resulting line of best fit is straight, the relationship is a direct variation. Proceed to Step 3.

2. If the line of best fit is curved, replot the graph to get a straight line as shown in Figure 9. If the first replotting results in a new curved line, draw yet another graph to obtain a straight line.

3. Determine the slope and y-intercept of the straight line on the graph. Substitute the values into the slope/y-intercept form of the equation that corresponds to the variables plotted on the graph with the straight line.

4. Check the equation by substituting original data points.

5. If required, use the equation (or the straight-line graph) to give examples of interpolation and extrapolation.

Example 2

Use regular graphing techniques to derive the equation relating the data given in Table 3.

<table>
<thead>
<tr>
<th>t (s)</th>
<th>0.00</th>
<th>2.00</th>
<th>4.00</th>
<th>6.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>v (m/s [E])</td>
<td>10.0</td>
<td>15.0</td>
<td>20.0</td>
<td>25.0</td>
</tr>
</tbody>
</table>

Solution: Figure 10 is the graph that corresponds to the data in Table 3. The line is straight and has the following slope:

\[
\text{slope} = \frac{\Delta v}{\Delta t} = \frac{25.0 \text{ m/s [E]} - 10.0 \text{ m/s [E]}}{6.00 \text{ s} - 0.00 \text{ s}}
\]

\[
\text{slope} = 2.50 \text{ m/s}^2 [\text{E}]
\]

The y-intercept is 10.0 m/s [E]. Using \(y = mx + b\), the equation is

\[
v = 2.50 \text{ m/s}^2 [\text{E}] (t) + 10.0 \text{ m/s [E]}
\]

Verify the equation by substituting \(t = 4.00 \text{ s}\):

\[
v = 2.50 \text{ m/s}^2 [\text{E}] (4.00 \text{ s}) + 10.0 \text{ m/s [E]}
\]

\[
= 10.0 \text{ m/s [E]} + 10.0 \text{ m/s [E]}
\]

\[
v = 20.0 \text{ m/s [E]}
\]

The equation is valid.

Using \(t = 3.20 \text{ s}\) as an example of interpolation,

\[
v = 2.50 \text{ m/s}^2 [\text{E}] (3.20 \text{ s}) + 10.0 \text{ m/s [E]}
\]

\[
= 8.00 \text{ m/s [E]} + 10.0 \text{ m/s [E]}
\]

\[
v = 18.0 \text{ m/s [E]}
\]
Example 3

Use regular graphing techniques to derive the equation for the data in the first two rows of Table 4.

Table 4 Acceleration–Mass Data

<table>
<thead>
<tr>
<th>m (kg)</th>
<th>2.0</th>
<th>4.0</th>
<th>6.0</th>
<th>8.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \ddot{a} ) (m/s² [E])</td>
<td>4.0</td>
<td>2.0</td>
<td>1.3</td>
<td>1.0</td>
</tr>
<tr>
<td>( \frac{1}{m} ) (kg⁻¹)*</td>
<td>0.50</td>
<td>0.25</td>
<td>0.167</td>
<td>0.125</td>
</tr>
</tbody>
</table>

* The third row is for the redrawn graph of the relationship.

Solution: Figure 11(a) is the graph of the data given in the first two rows of the table. Figure 11(b) shows the replotted graph with \( m \) replaced by \( \frac{1}{m} \), which produces a straight line.

The slope of the straight line is

\[
\text{slope} = \frac{\Delta \ddot{a}}{\Delta \left( \frac{1}{m} \right)}
\]

\[
= \frac{3.2 \text{ m/s}^2 [E] - 1.6 \text{ m/s}^2 [E]}{0.40 \text{ kg}^{-1} - 0.20 \text{ kg}^{-1}}
\]

\[
= 8.0 \text{ kg·m/s}^2 [E]
\]

The slope is 8.0 kg·m/s² [E], which can also be written 8.0 N [E].

The y-intercept is 0. Using \( y = mx + b \), the equation is

\[
\ddot{a} = 8.0 \text{ kg·m/s}^2 [E] \times \frac{1}{m}
\]

or \( \ddot{a} = \frac{8.0 \text{ N [E]}}{m} \)

Verify the equation by substituting \( m = 6.0 \) kg:

\[
\ddot{a} = \frac{8.0 \text{ kg·m/s}^2 [E]}{6.0 \text{ kg}}
\]

\[
\ddot{a} = 1.3 \text{ m/s}^2 [E]
\]

The equation is valid.

We can use this equation to illustrate extrapolation; for example, when the mass is 9.6 kg, the acceleration is

\[
\ddot{a} = \frac{8.0 \text{ kg·m/s}^2 [E]}{9.6 \text{ kg}}
\]

\[
\ddot{a} = 0.83 \text{ m/s}^2 [E]
\]

The acceleration is 0.83 m/s² [E].
**GRAPHING DATA ELECTRONICALLY**

You can use a graphing calculator or a computer-graphing program for several purposes, including determining the roots of an equation and analyzing linear functions, quadratic functions, trigonometric functions, and conic functions. You can also create a graph of given or measured data, and determine the equation relating the variables plotted or solve two simultaneous equations with two unknowns.

**Graphing calculators:** Examples 4, 5, and 6 show different ways to program a TI-83 graphing calculator. If you have a different graphing calculator, refer to its instruction manual for detailed information about solving equations.

---

**Example 4**

A ball is tossed vertically upward with an initial speed of 9.0 m/s. At what times after its release will the ball pass a position 3.0 m above the position where it was released? (Neglect air resistance.)

**Solution:** Defining upward as the positive direction and using magnitudes only, the given quantities are \( \Delta d = 3.0 \text{ m} \), \( v = 9.0 \text{ m/s} \), and \( a = -9.8 \text{ m/s}^2 \). The constant acceleration equation for displacement is

\[
\Delta d = v\Delta t + \frac{1}{2}a\Delta t^2
\]

\[
3.0 = 9.0\Delta t - 4.9\Delta t^2
\]

\[
4.9\Delta t^2 - 9.0\Delta t + 3.0 = 0
\]

To solve for \( \Delta t \), we can use the quadratic formula and enter the data into the calculator. The equation is in the form

\[
Ax^2 + Bx + C = 0,
\]

where \( A = 4.9 \), \( B = -9.0 \), and \( C = 3.0 \).

1. Store the coefficients A and B, and the constant C in the calculator:
   - \([\text{4.9 STO} \text{ ALPHA} \text{]}]
   - \([\text{ALPHA} \text{]}]
   - \([\text{-9 STO} \text{ ALPHA} \text{]}]
   - \([\text{ALPHA} \text{]}]
   - \([\text{3 STO} \text{ ALPHA} \text{]}]
   - \([\text{ENTER} \text{]}]

2. Enter the expression for the quadratic formula:

\[
-\frac{b \pm \sqrt{b^2 - 4ac}}{2a}
\]

- \([\text{ALPHA} \text{]} \text{+] [\text{4 STO} \text{ ALPHA} \text{]}] \text{/[} \text{ALPHA} \text{]} \text{/[}\text{2}] \text{]} \]

3. Press \([\text{ENTER} \text{]}\) to determine one solution for the time.

To determine the other solution, the negative must be used in front of the discriminant. The answers are 1.4 s and 0.44 s.

---

**Example 5**

Graph the function \( y = \cos x \) for \( 0^\circ \leq x \leq 360^\circ \).

**Solution:**

1. Put the calculator in degree mode:
   - \([\text{MODE} \rightarrow \text{Degree} \rightarrow \text{ENTER} \text{]}\]

2. Enter \( y = \cos x \) into the equation editor:
   - \( Y = \cos\{x, 0, 2\}\)

3. Adjust the window so that it corresponds to the given domain:
   - \([\text{WINDOW} \rightarrow X_{\min} = 0, X_{\max} = 360, X_{\text{interval}} = 90 \text{ (for an interval of } 90^\circ \text{ on the } x\text{-axis)}, Y_{\min} = -1, \text{ and } Y_{\max} = 1 \text{.}

4. Graph the function using the ZoomFit:
   - \([\text{ZOOM} \[\text{0} \text{]}\]

Consider an ellipse, which is important in physics because it is the shape of the orbits of planets and satellites. The standard form of the equation of an ellipse where the centre is the origin and the major axis is along the \( x \)-axis is

\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,
\]

where \( a > b \).

The vertices of the ellipse are at \((a, 0)\) and \((-a, 0)\), as shown in Figure 12.

**Figure 12 An ellipse**

---

**Example 6**

Use the “Zap-a-Graph” feature to plot an ellipse centred on the origin of an \( x \)-\( y \) graph, and determine the effect of changing the parameters of the ellipse.

**Solution:**

1. Choose Ellipse from the Zap-a-Graph menu:
   - \([\text{DEFINE} \rightarrow \text{Ellipse} \text{]}\]

2. Enter the parameters of the ellipse (e.g., \( a = 6 \) and \( b = 4 \)), then plot the graph.

3. Alter the ellipse by choosing Scale from the Grid menu and entering different values.
Graphing on a spreadsheet: A spreadsheet is a computer program that can be used to create a table of data and a graph of the data. It is composed of cells indicated by a column letter (A, B, C, etc.) and a row number (1, 2, 3, etc.). In Figure 13, B1 and C3 are examples of cells. Each cell can hold a number, a label, or an equation.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A1</td>
<td>B1</td>
<td>C1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>A2</td>
<td>B2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>A3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 13 Spreadsheet cells

To create a data table and plot the corresponding graph, you can follow the steps outlined in the next example.

Example 7

An object is moving with an initial velocity of 5.0 m/s [E] and a constant acceleration of 4.0 m/s² [E]. Set up a spreadsheet for the relationship \( \Delta d = v_0 \Delta t + \frac{1}{2} a \Delta t^2 \). Plot a graph of the data from \( t = 0 \) s to \( t = 8.0 \) s at intervals of 1.0 s.

Solution:
1. Access the spreadsheet, and label cell A1 the independent variable, in this case \( t \), and cell B1 the dependent variable, in this case \( \Delta d \).
2. Enter the values of \( t \) from 0 to 8.0 in cells A2 to A10. In cell B2 enter the right side of the equation in the following form: \( =5^*A2+(1/2)^*4^*A2^*A2 \) where \(^*\) represents multiplication.
3. Use the cursor to select B2 down to B10 and choose the Fill Down command, or right drag cell B1 down to B10 to copy the equation to each cell.
4. Command the program to graph the values in the data table (e.g., choose Make Chart, depending on the program).

A spreadsheet can be used to solve a system of two simultaneous equations, which can occur, for example, when analyzing elastic collisions (discussed in Section 5.3). In a one-dimensional elastic collision, the two equations involved simultaneously are

\[
m_1 v_1' + m_2 v_2' = m_1 v_1 + m_2 v_2
\]

and

\[
\frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2
\]

If the known quantities are \( m_1, m_2, v_1, \) and \( v_2 \), then the unknown quantities are \( v_1' \) and \( v_2' \). To determine the solution to these unknowns, rewrite the equations so that one unknown is isolated and written in terms of all the other variables. In this example, enter \( v_1' \) in cell A1, enter the first \( v_2' \) based on the equation for the law of conservation of momentum in cell B1, and enter the second \( v_2' \) (call it \( v_2'' \)) based on the equation for the law of conservation of energy in cell C1. Then proceed to enter the data according to the previous example, using reasonable values for the variables. Plot the data on a graph, and determine the intersection of the two resulting lines. That intersection is the solution to the two simultaneous equations.

A5.6 Unit Analysis

Unit analysis is a useful tool to confirm that a calculation has been performed correctly or to convert units. Three quantities that are commonly measured in physics are mass, \( m \), distance, \( \Delta d \), and time, \( \Delta t \). Note that the units of these measurements are all base units—kilogram (kg), metre (m), and second (s), respectively. In unit analysis, all units are expressed as base units. Derived units can be written in terms of SI base units and, thus, base dimensions. For example, the newton has base units of kg·m/s², or dimensions of \([\text{M}][\text{L}][\text{T}^{-2}]\). Appendix B contains a list of derived units.

After a while, unit analysis will become second nature. Consider a situation where you solve an equation in which \( \Delta t \) is the unknown, and your final answer is \( \Delta t = 2.1 \) kg. You know that something clearly has gone wrong, since time is measured in seconds (s). Check your calculation to see where the error was made. It is important to note that while unit analysis can clearly point out when you have made an error, units that work out correctly do not necessarily indicate that your answer is correct. We can use unit analysis to confirm that the equation \( \Delta d = v_1 t + \frac{1}{2} a \Delta t^2 \) is mathematically valid.

To perform unit analysis, insert the appropriate units into the equation that you are analyzing. Place all units in square brackets, and ignore any fractions in the equation that you are analyzing.

\[
[\Delta d] = [v_1][t] + \frac{1}{2} [a][\Delta t]^2
\]

\[
[m] = \frac{m}{s} [s] + \frac{m}{s^2} [s^2]
\]

\[
[m] = [m] + [m]
\]

\[
[m] = [m]
\]

Since both sides of the equation have the same units, metres (m), unit analysis has shown that this equation is mathematically valid.
The reason we ignore fractions in dimensional analysis is because fractions have no units. They are "dimensionless quantities." Dimensionless quantities include:

- all plain numbers (4, $\pi$, etc.) and counted quantities (12 people, 5 cars, etc.)
- angles (although angles have units)
- cycles
- trigonometric functions
- exponential functions
- logarithms

You can also use unit analysis to convert from one unit to another. For example, to convert 95 km/h to m/s, kilometres must be changed to metres, and hours must be changed to seconds. Using the conversions 1 km = 1000 m, 1 h = 60 min, and 1 min = 60 s, the following ratios can be written:

$$\frac{1000 \text{ m}}{1 \text{ km}}; \quad \frac{1 \text{ h}}{60 \text{ min}}; \quad \frac{1 \text{ min}}{60 \text{ s}} = 1$$

To convert from km/h to m/s we will use these three ratios strategically to cancel units that we do not want, and to keep units that we do want (m and s). These ratios can be inverted if necessary:

$$\frac{95 \text{ km}}{1 \text{ h}} = \frac{95 \text{ km}}{1 \text{ km}} \times \frac{1000 \text{ m}}{1 \text{ km}} \times \frac{1 \text{ h}}{60 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ s}}$$

Therefore,

$$\frac{95 \text{ km}}{1 \text{ h}} = \frac{95000 \text{ m}}{3600 \text{ s}} = 26.4 \text{ m/s}, \text{ or } 26 \text{ m/s (to two significant digits)}$$

### Example 8

A 2100 g object experiences a net force of magnitude 38.2 N. Determine the magnitude of the object’s acceleration.

**Given**: $m = 2100 \text{ g}$; $F_{\text{net}} = 38.2 \text{ N}$

**Required**: $a$

**Analysis**: First, convert grams to kilograms. Then use the equation $F_{\text{net}} = ma$ to calculate acceleration.

**Solution**: $m = \frac{2100 \text{ g}}{1000 \text{ g}} = 2.1 \text{ kg}$

$F_{\text{net}} = ma$

$a = \frac{F_{\text{net}}}{m}$

$a = \frac{38.2 \text{ N}}{2.1 \text{ kg}}$

$a = 18 \text{ m/s}^2$

It is more common to express acceleration using the SI units of metres per second squared, so we will use unit analysis to convert the units:

$$F_{\text{net}} = ma$$

$$[N] = \left[ \frac{\text{kg} \cdot \text{m}}{\text{s}^2} \right]$$

$a = \frac{18 \text{ N}}{\text{kg}}$

$$a = \frac{18 \text{ kg} \cdot \text{m}}{\text{s}^2}$$

$a = 18 \text{ m/s}^2$

**Statement**: The acceleration is $18 \text{ m/s}^2$.

### A5.7 Vectors

Several quantities in physics are vector quantities—quantities that have both magnitude (size) and direction. Understanding and working with vectors is crucial in solving many physics problems.

**VECTOR SYMBOLS**

A vector is represented in a vector scale diagram by an arrow, or a directed line segment. The length of the arrow is proportional to the magnitude (size) of the vector, and the direction is the same as the direction of the vector. The tail of the vector is the initial point, and the tip of the vector is the end with the arrowhead. When the vector is drawn to scale, the scale should be indicated on the diagram, as should the direction north (Figure 14).

<table>
<thead>
<tr>
<th>(a)</th>
<th>(b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>\vec{a}</td>
</tr>
<tr>
<td>scale: 1 cm = 10 m</td>
<td>scale: 1 cm = 5 m/s²</td>
</tr>
</tbody>
</table>

**Figure 14** Examples of vector quantities: (a) displacement vector; (b) acceleration vector

In this text, a vector quantity is indicated by an arrow above the letter representing the vector (for example, $\vec{A}$, $\vec{F}$, $\vec{p}$). The magnitude of a vector, for example, $|\vec{A}|$, is indicated by the letter without the vector arrow, in this example, $A$. The magnitude is always positive.
DIRECTIONS OF VECTORS
The directions of vectors are indicated in square brackets following the magnitude and units of the measurement. The four compass directions—east, west, north, and south—are indicated as [E], [W], [N], and [S]. Other examples are [down], [forward], [11.5° below the horizontal], [toward Earth’s centre], and [W 24° N].

Figure 15 shows a vector drawn in the northwest quadrant of a Cartesian coordinate system. We designate the direction of this vector as [W 24° N]. This can be read as, “the vector initially pointed west, and was then rotated 24° toward north.” Since the complementary angle of 24° is 66°, this direction can also be expressed as [N 66° W]. Both of these directions are equally correct.

\[ \Delta d = 15 \text{ m [E 30° N]} \]

\[ \Delta d_x = \Delta d \cos 30° \]
\[ \Delta d_y = \Delta d \sin 30° \]

\[ \Delta d_x = (15 \text{ m}) \cos 30° \]
\[ \Delta d_y = (15 \text{ m}) \sin 30° \]
\[ \Delta d_x = 13 \text{ m [E]} \]
\[ \Delta d_y = 7.5 \text{ m [N]} \]

VECTORS

\[ \cos 30° = \frac{\Delta d_x}{\Delta d} \]
\[ \sin 30° = \frac{\Delta d_y}{\Delta d} \]

We can use trigonometry to determine the magnitude of each component vector: When possible, draw \( x \)-components along the \( x \)-axis; this will ensure that each \( x \)-component vector will have a cosine term. Similarly, each \( y \)-component vector will have a sine term. Depending on the problem being solved, \( \theta \) will not always be the angle between the \( x \)-axis and the displacement. Sometimes, \( \theta \) will be between the \( y \)-axis and the displacement. Therefore, always consider which component is opposite \( \theta \) and which one is adjacent to \( \theta \) to determine the components.

**VECTOR ADDITION**
In arithmetic, \( 3 + 3 \) always equals 6. This is not always true when adding vector quantities. Vector addition must take into consideration the directions of the vectors. Here are some general guidelines regarding vector addition:

- **Vector addition is commutative.** The order of addition does not matter: \( \vec{A} + \vec{B} = \vec{B} + \vec{A} \)
- **Vector addition is associative.** If more than two vectors are added, it does not matter how they are grouped: \( (\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C}) \)

In this text we model three methods to add vectors. Vector addition by scale diagram requires the use of a protractor and metric ruler to add vectors. The sine and cosine laws method only works when adding two vectors at a time, but the result is more accurate than that of a scale diagram. Vector addition by components is a purely algebraic method for adding vectors.

The Scale Diagram Method
To add vector quantities by scale diagram, the arrows representing the vectors are drawn to scale and are joined tip to tail. The resultant vector is drawn starting at the tail of the first vector and ending at the tip of the last vector added. Vector addition by scale diagram tends to be very simple for one-dimensional problems. We will solve a one-dimensional and a two-dimensional problem in the following examples.

**COMPONENTS OF VECTORS**
Any two-dimensional vector can be broken down into two perpendicular components. Figure 16 shows a displacement vector \( \Delta \vec{d} = 15 \text{ m [E 30° N]} \) drawn on a Cartesian coordinate system.

This vector has been broken down into \( x \)- and \( y \)-components such that the \( x \)-component is drawn along the \( x \)-axis of the Cartesian coordinate system. When the two component vectors \( \Delta d_x \) and \( \Delta d_y \) are joined tip to tail as shown in the diagram, they will give the resultant vector \( \Delta \vec{d} = 15 \text{ m [E 30° N]} \). The directions of these two component vectors are clear from the diagram.
Example 9

Add the following two displacements using the scale diagram method:

\[ \Delta \vec{d}_1 = 3.0 \text{ m [E]}, \Delta \vec{d}_2 = 5.0 \text{ m [W]} \]

Note: When drawing a vector scale diagram, choose a scale that will produce a diagram that is approximately one-half to one full page in size. Be sure to indicate the scale and direction due north in your diagram.

**Given:** \( \Delta \vec{d}_1 = 3.0 \text{ m [E]}; \Delta \vec{d}_2 = 5.0 \text{ m [W]} \)

**Required:** \( \Delta \vec{d}_t \)

**Analysis:** Use the scale 1 cm = 1 m.

\[ \Delta \vec{d}_t = \Delta \vec{d}_1 + \Delta \vec{d}_2 \]

**Solution:**

\[ \Delta \vec{d}_t = 3.0 \text{ m [E]} + 5.0 \text{ m [W]} \]

**Figure 17** shows the total displacement, represented by the vector \( \Delta \vec{d}_t \). From this diagram we can see that the total displacement is represented by a vector 2.0 cm long pointing due west. Applying our scale we get \( \Delta \vec{d}_t = 2.0 \text{ m [W]} \).

**Statement:** The total displacement is 2.0 m [W].

Example 10

Add the following two displacement vectors using the scale diagram method:

\[ \Delta \vec{d}_1 = 4.0 \text{ m [E]}, \Delta \vec{d}_2 = 5.0 \text{ m [W 30° S]} \]

**Solution:** Use the scale 1 cm = 1 m. **Figure 18** shows these two vectors joined tip to tail in a scale diagram. The length of the total displacement is measured to be 4.6 cm. Applying our scale and using our protractor to measure the angle for the direction of this displacement vector, we get \( \Delta \vec{d}_t = 4.6 \text{ m [E 71° S]} \).

You can use the cosine and sine laws method with Example 10 and get the same results.

**Adding Vectors Algebraically**

While vector addition by scale diagram is effective, it is not terribly precise. It can also become quite complex when adding more than two vectors. A purely algebraic method of adding vectors is far more precise. This method uses trigonometry and the Pythagorean theorem.

Example 11

Consider two vector forces acting on a single object. Determine the vector sum of these two forces.

\[ \vec{F}_1 = 10.5 \text{ N [S]}, \vec{F}_2 = 14.0 \text{ N [W]} \]

**Given:** \( \vec{F}_1 = 10.5 \text{ N [S]}; \vec{F}_2 = 14.0 \text{ N [W]} \)

**Required:** \( \vec{F}_t \)

**Analysis:** \( \vec{F}_t = \vec{F}_1 + \vec{F}_2 \)

**Solution:** **Figure 19** shows the two force vectors joined tip to tail. Note that this is only a sketch; it is not drawn to scale. We can determine the magnitude of the resultant vector \( \vec{F}_t \) using the Pythagorean theorem.

\[ \vec{F}_1 = 10.5 \text{ N [S]} \]

\[ \vec{F}_2 = 14.0 \text{ N [W]} \]

**Figure 19** Determining the net force

\[ \vec{F}_t = \vec{F}_1 + \vec{F}_2 \]

\[ F_t = \sqrt{F_1^2 + F_2^2} \]

\[ = \sqrt{(10.5 \text{ N})^2 + (14.0 \text{ N})^2} \]

\[ = 17.5 \text{ N} \]

To determine the direction of the resultant vector, use the tangent function:

\[ \tan \theta = \frac{F_2}{F_1} \]

\[ \theta = \tan^{-1} \left( \frac{14.0 \text{ N}}{10.5 \text{ N}} \right) \]

\[ \theta = 53.1° \]

**Statement:** The total force acting on the object is 17.5 N [S 53.1° W].
Often we are required to add vectors that are not perpendicular to each other. We can still do this algebraically. The method is called vector addition by components. The goal is to take a question and convert it into the sample problem that we have just solved, that is, to take a general two-dimensional vector problem and convert it into a problem where we have two perpendicular vectors. To add any number of vectors by components, use the following steps:

1. Draw a Cartesian coordinate system.
2. Draw each vector to be added on the Cartesian coordinate system starting at the origin.
3. Break each vector down into x- and y-components such that the x-component is drawn along the x-axis.
4. Determine the sum of the x-components by adding all the individual x-components.
5. Determine the sum of the y-components by adding all the individual y-components.
6. Draw a sketch showing the overall x- and y-component vectors joined tip to tail.
7. Determine the magnitude and direction of the resultant vector by using the Pythagorean theorem and tangent function.

**Example 12**

Determine the vector sum of the two following displacements:

\[ \Delta \vec{d}_1 = 25 \text{ m} \ [N \ 40.0^\circ \ W], \ \Delta \vec{d}_2 = 35 \text{ m} \ [S \ 20.0^\circ \ E] \]

**Given:** \( \Delta \vec{d}_1 = 25 \text{ m} \ [N \ 40.0^\circ \ W] \); \( \Delta \vec{d}_2 = 35 \text{ m} \ [S \ 20.0^\circ \ E] \)

**Required:** \( \Delta \vec{d}_t \)

**Analysis:** \( \Delta \vec{d}_t = \Delta \vec{d}_1 + \Delta \vec{d}_2 \)

**Solution:** \( \Delta \vec{d}_t = \Delta \vec{d}_1 + \Delta \vec{d}_2 \)

**Figure 20** shows our two displacement vectors drawn on a Cartesian coordinate system. Each vector is drawn from the origin and has an x-component, which is drawn along the x-axis.

![Figure 20](image)

**Figure 20** shows our two displacement vectors drawn on a Cartesian coordinate system. Each vector is drawn from the origin and has an x-component, which is drawn along the x-axis.

**Calculate the vector sum of the x-components.**

\[
\Delta d_{tx} = \Delta d_{tx1} + \Delta d_{tx2} \\
= \Delta d_1 \cos \theta_1 + \Delta d_2 \cos \theta_2 \\
= (25 \text{ m})(\cos 50.0^\circ) \ [W] + (35 \text{ m})(\cos 70.0^\circ) \ [E] \\
= 16.1 \text{ m} \ [W] + 12.0 \text{ m} \ [E] \\
= 16.1 \text{ m} \ [W] - 12.0 \text{ m} \ [W] \\
\Delta d_{tx} = 4.10 \text{ m} \ [W] \text{ (one extra digit carried)}
\]

**Calculate the vector sum of the y-components.**

\[
\Delta d_{ty} = \Delta d_{ty1} + \Delta d_{ty2} \\
= \Delta d_1 \sin \theta_1 + \Delta d_2 \sin \theta_2 \\
= (25 \text{ m})(\sin 40.0^\circ) \ [N] + (35 \text{ m})(\sin 70.0^\circ) \ [S] \\
= 19.2 \text{ m} \ [N] + 32.9 \text{ m} \ [S] \\
= -19.2 \text{ m} \ [S] + 32.9 \text{ m} \ [S] \\
\Delta d_{ty} = 13.7 \text{ m} \ [S] \text{ (one extra digit carried)}
\]

**Figure 21** shows a sketch of the overall x- and y-component vectors joined tip to tail.

![Figure 21](image)

**Figure 21** shows a sketch of the overall x- and y-component vectors joined tip to tail.

We can now use the Pythagorean theorem to determine the magnitude of the resultant vector and the tangent function to determine its direction.

\[
\Delta d_t = \sqrt{\Delta d_{tx}^2 + \Delta d_{ty}^2} \\
\Delta d_t = \sqrt{4.10^2 + 13.7^2} \\
\phi = \tan^{-1}\left(\frac{13.7}{4.10}\right) \\
\Delta d_t = 14 \text{ m} \\
\phi = 73^\circ
\]

**Statement:** The vector sum of the displacements is 14 m [W 73° S].
THE SCALAR OR DOT PRODUCT OF TWO VECTORS

The scalar product of two vectors is equal to the product of
their magnitudes and the cosine of the angle between the
vectors. The scalar product is also called the dot product
because a dot can be used to represent the product symbol.
An example of a scalar product is the equation for the work
W done by a net force \( \vec{F} \) that causes an object to move by a
displacement \( \Delta \vec{d} \) (Section 4.1).

\[
W = \vec{F} \cdot \Delta \vec{d} = F \Delta d \cos \theta
\]

or

\[
W = (F \cos \theta) \Delta d
\]

Thus, the defining equation of the scalar (or dot) product of
vectors \( \vec{A} \) and \( \vec{B} \) is

\[
\vec{A} \cdot \vec{B} = AB \cos \theta
\]

where \( \theta \) is the angle between \( \vec{A} \) and \( \vec{B} \), \( A \) is the magnitude
of \( \vec{A} \), and \( B \) is the magnitude of \( \vec{B} \). Notice that \( \vec{A} \) and \( \vec{B} \) do
not represent the same quantities.

A scalar product can be represented in a diagram as
shown in Figure 22 in which an applied force \( \vec{F}_a \) is at an
angle \( \theta \) to the displacement of the object being pulled (with
negligible friction).

![Figure 22 Assuming that there is negligible friction, the work done by \( \vec{F}_a \) on the cart in moving it a displacement \( \Delta \vec{d} \) is the scalar
product, \( F_a \cos \theta \Delta d \).](image)

THE VECTOR OR CROSS PRODUCT

OF TWO VECTORS

The vector product of two vectors has a magnitude equal to
the product of the magnitudes of the two vectors and the
sine of the angle between the vectors. The vector product
is also called the cross product because a "\( \times \)" is used to
represent the product symbol. Thus, for vectors \( \vec{A} \) and \( \vec{B} \),
the vector product \( \vec{C} \) is defined by the following equation:

\[
\vec{C} = \vec{A} \times \vec{B}
\]

where the magnitude is given by \( C = |\vec{C}| = |AB \sin \theta| \), and
the direction is perpendicular to the plane formed by \( \vec{A} \)
and \( \vec{B} \). However, there are two distinct directions that are
perpendicular to the plane formed by \( \vec{A} \) and \( \vec{B} \). To determine
the correct direction, you can use the following rule,
illustrated in Figure 23:

- Right-hand rule for the vector product: When the
fingers of the right hand move from \( \vec{A} \) toward \( \vec{B} \), the
outstretched thumb points in the direction of \( \vec{C} \).

![Figure 23 The right-hand rule to determine the direction of the vector resulting from the vector product \( \vec{C} = \vec{A} \times \vec{B} \).](image)

The vector (or cross) product has the following properties:

- The order in which the vectors are multiplied matters
because \( \vec{A} \times \vec{B} = - \vec{B} \times \vec{A} \). (Use the right-hand
rule to verify this.)
- If \( \vec{A} \) and \( \vec{B} \) are parallel, \( \theta = 0^\circ \) or \( 180^\circ \) and \( \vec{A} \times \vec{B} = 0 \)
because \( \sin 0^\circ = \sin 180^\circ = 0 \). Thus, \( \vec{A} \times \vec{A} = 0 \).
- If \( \vec{A} \) is perpendicular to \( \vec{B} \) (\( \theta = 90^\circ \)), then
\( |\vec{A} \times \vec{B}| = AB \) because \( \sin 90^\circ = 1 \).
- The vector product obeys the distributive law; that is,
\( \vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C} \).

LEARNING TIP

Alternative Notation

In advanced physics textbooks, vectors are sometimes written in
boldface rather than with an arrow above the quantity. Thus,
you may determine the dot product and the cross product written
as follows:

\[
\vec{A} \cdot \vec{B} = AB \cos \theta
\]

\[
\vec{A} \times \vec{B} = AB \sin \theta
\]
Choosing Appropriate Career Pathways

Often, one of the most difficult tasks in high school is deciding what career path to follow after graduation. The science skills and concepts presented in this book will be of benefit to many careers, whether you are planning a career in scientific research (such as research geneticist or astrophysicist) or in areas related to science (such as environmental lawyer, pharmaceutical sales representative, or electrician). The strong critical-thinking and problem-solving skills that are emphasized in science programs are a valuable asset for any career.

Career Links and Pathways

Throughout this textbook you will have many opportunities to explore careers related to your studies in physics. The Career Links icons in the text indicate that you can learn more about a career related to the text by going to the Nelson Science website. At the end of each chapter you will also find a Career Pathways feature that illustrates sample educational pathways for some of the careers related to the chapter.

It is wise to begin researching academic requirements as early as possible. Understanding the options available to pursue a particular career will help you make decisions on whether to attend university or college, and which program of study you should take. In addition, understanding the terminology used by universities and colleges will play an integral role in planning your future.

University and College Programs

Undergraduate university programs generally lead to a three-year general bachelor degree or a four-year honours bachelor degree. These degree designations begin with a “B” followed by the area of specialization; for example, a B.Sc. (Hons.) indicates an Honours Bachelor of Science degree. These degrees can lead to employment or to further education in Masters or Doctorate postgraduate programs. The length of postgraduate degrees generally varies from one to four years.

College programs typically fall into three categories: one-year certificates, two-year diplomas, and three-year advanced diplomas. Certificates and diplomas can lead directly to employment opportunities or to graduate certificate programs. In some programs, there are “transfer agreements” with universities, which allow college graduates to enter university programs with advanced standing toward a university degree.

Pathways in Physics

The Career Pathways graphic organizer illustrates examples of pathways to follow after high school. Certain pathways lead to careers after university, and others may lead to careers after college. Look at Figure 1 below to see three possible career pathways.

Student A wishes to become an atmospheric scientist and must complete the Grade 11 and 12 University Physics courses (along with other prerequisites) and enter an undergraduate university program. Student A must obtain a Bachelor of Science degree, and then continue on to further education in Masters and Doctorate programs before becoming an atmospheric scientist.

Student B wishes to become a Geographic Information System (GIS) analyst and must complete the Grade 11 University Physics course (along with other prerequisites) and enter an undergraduate university program. Student B must obtain a Bachelor of Arts degree, for example in Geography, and then continue on to a college for a graduate certificate program in GIS before becoming a GIS analyst. Student C could become a GIS analyst by entering a college certificate program, then completing the same graduate certificate program as student B.

Planning for Your Future

Planning ahead for your educational and career paths will provide a rewarding future. You should consult your guidance counsellors for specific advice on career planning and which courses you should take in high school. Take the time to research university and college websites for specific program information because these sites will provide the prerequisite information and, most often, career planning advice.