Lesson 3: Rotational Symmetry

**Goal**
Determine whether and how a shape can be turned to fit on itself.

**Prerequisite Skills/Concepts**
- Know the concept of rotation.
- Measure angles of rotation.

**Expectations**
- sort polygons according to [the number of lines of symmetry and] the order of rotational symmetry, through investigation using a variety of tools
- identify, perform, and describe, through investigation using a variety of tools, rotations of $180^\circ$ and clockwise and counterclockwise rotations of $90^\circ$, with the centre of rotation inside or outside the shape
- create and analyze designs made by reflecting, translating, and/or rotating a shape, or shapes, by $90^\circ$ or $180^\circ$

**Assessment for Feedback**

<table>
<thead>
<tr>
<th>What You Will See Students Doing...</th>
<th>Students will</th>
<th>When Students Understand</th>
<th>If Students Misunderstand</th>
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<tbody>
<tr>
<td>determine the rotational symmetry of a shape</td>
<td>Students will trace the shape and determine its rotational symmetry by rotating a cutout of that shape over the tracing to find how often the shape fits on itself more than once during one complete rotation.</td>
<td>Students will predict the rotational symmetry of a shape before rotating it.</td>
<td>Have students who have difficulty rotating the cutout about its centre use a pencil or thumb tack to pin it at its centre of rotation through the tracing. Remind them to mark one corner or tip of the cutout to help them count how many times the shape fits on itself more than once during one complete rotation.</td>
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<td>predict and determine the order of rotational symmetry for a shape</td>
<td>Students will determine the order of rotational symmetry for shapes and sort them accordingly.</td>
<td></td>
<td>Ask students who have difficulty predicting a shape’s rotational symmetry to consider the shape’s line symmetry as a clue.</td>
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<td>sort shapes according to their order of rotational symmetry</td>
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<td>Suggest that students record a tally mark for each time a given shape fits on itself during one complete rotation, and then compare their totals.</td>
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**Preparation and Planning**

**Pacing**
- 5–10 min Introduction
- 15–20 min Teaching and Learning
- 20–30 min Consolidation

**Materials**
- 3-D model sets (3–4/class)
- pattern blocks (1 set/pair)
- scissors (1/pair)
- tracing paper (several sheets/student)
- stickers (several/student)
- Venn diagrams (1/student)

**Masters**
- Venn Diagram 2, Masters Booklet p. 56
- Optional: Chapter 14 Mental Math p. 59

**Workbook**
- p. 126

**Vocabulary/ Symbols**
- rotational symmetry
- order of rotational symmetry

**Key Assessment of Learning Question**
- Question 5, Application of Learning

**Mathematical Processes**
- Selecting Tools and Strategies, Reasoning and Proving

**Meeting Individual Needs**

**Extra Challenge**
- Have students search their classroom for shapes having rotational symmetry and determine their order of rotational symmetry. Ask them to take photos or draw diagrams of the shapes, and to use them to make a set of posters on which shapes having the same order of rotational symmetry are displayed on individual posters.
- As an art project, have students develop designs having different orders of rotational symmetry.

**Extra Support**
- Give students a square pyramid, cube, and rectangular prism from the 3-D model set. Have them rotate each shape at eye level to see which ones look exactly the same after quarter turns, half turns, and whole turns. Ask students to then put a sticker on one corner of the base of the square pyramid and note the sticker’s position each time they rotate the shape a quarter turn. Have students trace the base of the pyramid on paper and record the position of the sticker after each quarter turn. Ask students to then repeat this process with the other two shapes, and then say what the order of rotational symmetry is for each shape.
Start the lesson by asking for a single volunteer to come to the front of the class. With the volunteer facing the class, tell students that you are now going to demonstrate rotation in 3-D by asking the volunteer to slowly rotate (turn) on the spot on which he or she is standing. Tell the class that they are to raise their hands when they see the volunteer’s complete face again. When everyone is ready, ask the volunteer to begin turning in stages with a short rest between each stage. As the hands go up, engage the class in a discussion along the following lines.

Sample Discourse
“How far around did the volunteer rotate before you could see his or her complete face again? Why was this true?”
• It was one complete rotation because a person only has one face.

“How can you find an object in the room that would look the same more than once during a rotation?”

Ask students to discover large, different-shaped objects such as a tissue box (cube or rectangular prism), a circular (cylinder or truncated cone) or rectangular waste basket (trapezoid prism), and an eraser. Have the volunteer horizontally rotate these objects slowly in front of the class where everyone can see them.

“What do you notice about how often these objects look the same as they are rotated?”
• A cubic tissue box looks the same after every quarter rotation.
• The eraser and rectangular tissue box look the same when they’re rotated half way around.
• A circular waste basket looks the same no matter how much or little you turn it, but one with a rectangular top and bottom has to be rotated half way around before it looks the same.

“So, different objects need to rotate different degrees before they look the same again. Why is that?”
• The objects all have different shapes.

“Based on what you have seen, which shapes need to rotate less to look the same?”
• Shapes that are more symmetrical, like the circular waste basket, need to rotate less.

Tell students that, in this lesson, they will find out whether more symmetrical 2-D shapes also need to rotate less to look the same.

Make sure that students understand that, while a shape with order of rotational symmetry 1 can fit on itself exactly once during a complete rotation, it does not have rotational symmetry.

Divide the class into pairs and have the pairs repeat Li Ming’s steps by first making two tracings of a square pattern block on paper. Then have them cut out one tracing and position it on top of the other one and mark the centre of rotation before following the rest of her steps to test her prediction.

Have the pairs complete the Reflecting questions next. Allow time for students to share their responses as a class.

Reflecting
These questions are designed to stimulate students to think about the sequence of steps needed to determine the rotational symmetry of a given shape, and to help them understand the “whys.” When the “whys” of a certain procedure are understood, it is much easier to retain the “how.”
Answers

1. For example, if you didn't put the sticker on, you wouldn't be able to tell how many times you had rotated the square. (Lesson 3 Answers continued on pp. 79–80)

Consolidation  20–30 min

Checking (Pairs)

For intervention strategies, refer to Meeting Individual Needs or the Assessment for Feedback chart.

3. In pairs, have students trace the pattern block piece of the same shape on paper and then experiment by rotating the piece on top of the tracing to determine whether it has rotational symmetry. Students who have difficulty might not be able to predict its order of rotational symmetry in a meaningful way, but should be able to follow Li Ming's steps to determine it.

Practising (Individual)

4.–6. These questions require students to follow Li Ming's same steps to determine the rotational symmetry of the given shapes. Ask students who have difficulty making accurate predictions to visualize what the shapes will look like after each leading edge is rotated into the position of the one in front of it until the shape completes a full rotation.

7. Have students determine the rotational symmetry and order of rotational symmetry for each shape before sorting the shapes with a Venn diagram.

8. Ask students to experiment with different examples of polygons that match the description in the question.

Related Questions to Ask

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<thead>
<tr>
<th>Ask</th>
<th>Possible Responses</th>
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<tr>
<td><strong>About Question 7:</strong> What will the order of rotational symmetry be for shapes that have rotational symmetry?</td>
<td>• Shapes with rotational symmetry will have order of rotational symmetry greater than 1.</td>
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<tr>
<td><strong>About Question 8:</strong> Do circular shapes have more rotational symmetry than polygons with straight edges?</td>
<td>• Circles have more rotational symmetry because they can fit on themselves many more times than polygons with straight edges during one complete rotation.</td>
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Closing (Whole Class)

Ask students to explain the rotational symmetry of a shape by drawing that shape in their journals and describing how they can determine its rotational symmetry using diagrams. Then ask them to summarize what they have learned by completing the following sentence: “One thing that I learned about rotational symmetry is….”
Assessment of Learning—What to Look for in Student Work…

Assessment Strategy: Written Answer
Application of Learning

Key Assessment Question 5
a) Predict the order of rotational symmetry for these shapes.
b) Check your prediction.


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<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
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<td>• demonstrates <strong>limited</strong> ability to transfer mathematical knowledge and skills to new contexts (e.g., has difficulty using rotational symmetry to predict and determine the order of symmetry for the shapes)</td>
<td>• demonstrates <strong>some</strong> ability to transfer mathematical knowledge and skills to new contexts (e.g., demonstrates some ability to use rotational symmetry to predict and determine the order of symmetry for the shapes)</td>
<td>• demonstrates <strong>considerable</strong> ability to transfer mathematical knowledge and skills to new contexts (e.g., uses rotational symmetry to predict and determine the order of symmetry for the shapes)</td>
<td>• demonstrates <strong>sophisticated</strong> ability to transfer mathematical knowledge and skills to new contexts (e.g., demonstrates sophisticated ability to use rotational symmetry to predict and determine the order of symmetry for the shapes)</td>
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Extra Practice and Extension
• You might assign any of the questions related to this lesson, which are cross-referenced in the chart below.

<table>
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<tr>
<th>Curious Math</th>
<th>Mid-Chapter Review</th>
<th>Skills Bank</th>
<th>Problem Bank</th>
<th>Chapter Review</th>
<th>Workbook</th>
<th>Nelson Web Site</th>
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</thead>
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At Home
• Have students find and list things around their homes that have rotational symmetry. For example, they could mention garbage cans, recycling boxes, paper towel rolls, shoeboxes, and so on. Ask them to sketch at least four of these items and determine their order of rotational symmetry.

Venn Diagram 2, Masters Booklet p. 56

Math Background
Rotational symmetry can be a difficult topic for students to master because the centre about which the shape should turn is not clearly defined. The steps offered in this lesson, that is first tracing the shape and then rotating a cutout of that shape on top of that tracing, make determining rotational symmetry easier for students. Students can easily confuse rotational symmetry with its order. While rotational symmetry is defined by whether a shape can fit on itself more than once, the order of rotational symmetry is defined by the number of times a shape fits on itself in a complete rotation. Shapes that fit on to their shapes only once during a complete rotation are said to not have rotational symmetry, but to have order of rotational symmetry 1.

Optional: Chapter 14 Mental Math, p. 59