Lesson 6: Percents as Special Ratios

CHAPTER 12

Percents as Special Ratios

STUDENT BOOK PAGES 368–369

Direct Instruction

Goal

Understand the meaning of percent.

Prerequisite Skills/Concepts

• Relate fractions, decimals, and ratios.

Expectations

• demonstrate an understanding of relationships involving percent, ratio [], and unit rate
• determine and explain, through investigation using concrete materials, drawings, and calculators, the relationships among fractions, decimal numbers, and percents

Assessment for Feedback

<table>
<thead>
<tr>
<th>Students will</th>
<th>When Students Understand</th>
<th>If Students Misunderstand</th>
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<tbody>
<tr>
<td>• demonstrate an understanding of percent</td>
<td>• Students will use grids correctly to relate ratios to equivalent fractions to percents.</td>
<td>• Students may fail to grasp that fractions, ratios, and percents are all ways of comparing a part to the whole. Provide extra practice in identifying the part and whole in fractions, part to whole ratios, and percents. Clarify that a fraction or percent cannot be used to represent a part to part ratio. For example, tell students that Clara has 4 candies, 1 red and 3 white. Have them write the ratio red:white. Ask them what kind of ratio this is (part to part). Ask why, in this situation, the fraction 1/3 is not equal to the ratio 1:3 (because the 3 in the fraction stands for 3 parts in the whole, and there are 4 parts in the set of candies; 1/4 of the candies are red).</td>
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Preparation and Planning

Pacing

<table>
<thead>
<tr>
<th>5–10 min Introduction</th>
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<tbody>
<tr>
<td>15–20 min Teaching and Learning</td>
</tr>
<tr>
<td>20–30 min Consolidation</td>
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</table>

Materials

• calculators (1/student or pair)
• Optional: 10-by-10 grid on acetate for demonstration (2 grids)

Masters

• Optional: Chapter 12 Mental Math p. 70
• 10-by-10 Grid, Masters Booklet p. 36

Workbook

p. 109

Vocabulary/Symbols

percent, %

Key Assessment of Learning Question

Question 6, Knowledge and Understanding

Mathematical Processes

Representing, Communicating, Connecting

Meeting Individual Needs

Extra Challenge

• Students have likely heard the expression “give it 110%.” Challenge them to explain why this expression is mathematically unsound.

Extra Support

• Provide students with a list of all the factors of 100, explaining what the list represents. (Providing the factors for students will enable you to keep the emphasis on determining equivalent hundredths.) Then provide them with a selection of fractions with 2, 4, 5, 10, 20, 25, and 50 in the denominator, and part to whole ratios with one of these numbers as the whole, and have them practice creating equivalent fractions with 100 in the denominator, and then determining the percent. They could record their work in a chart such as the one below.

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Factors of 100</th>
<th>Equivalent Fraction or Ratio</th>
<th>Percent</th>
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<tbody>
<tr>
<td>1/2</td>
<td>2 × 50 = 100</td>
<td>50/100</td>
<td>50%</td>
</tr>
<tr>
<td>1 : 4</td>
<td>4 × 25 = 100</td>
<td>25/100</td>
<td>25%</td>
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</tbody>
</table>

Then provide another assortment of fractions and ratios, some of which can be expressed as an equivalent fraction with 100 in the denominator and some of which cannot. Have students determine which are which and explain why.
Introduction (Whole Class) 5–10 min

Briefly review equivalent fractions and their relationship to equivalent ratios. On the board, draw three squares of equal size. Divide one into halves, and below it write \( \frac{1}{2} \).

Ask for volunteers to use the other two squares to create representations of fractions equivalent to \( \frac{1}{2} \) (e.g., \( \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \ldots \)).

Recall how equivalent fractions represent the same part of a whole (or a set). Then write the ratio 1 : 2 beneath the first square, and have volunteers write the corresponding ratios below the other two squares (e.g., 2 : 4, 3 : 6, 4 : 8, etc.).

Recall that equivalent ratios represent the same comparison.

Tell students that, in this lesson, they will be learning about a special ratio that makes it easy to compare parts of a whole or set.

Teaching and Learning (Whole Class/Pairs) 15–20 min

Distribute 10-by-10 grids to students.

Ask students to turn to Student Book page 368 and, as a class, read the central question and James’s Solution. Go over the definition of percent and ask students to share examples of percent in real life. Encourage students to think of percent as another way of saying hundredths.

Have pairs of students go over James’s Solution. Instruct them to outline and colour the ratio and the fraction on their 10-by-10 grids. Ask one student pair to share their work on the overhead. Ensure students understand why 25 squares were outlined and why 4 squares were coloured for the Atlantic locations, and why 20 squares were outlined and 7 squares were coloured for the Pacific locations.

When students have completed the activity, you may want to show them how to use their calculators to turn a fraction into decimal hundredths to determine percent. Choose a simple fraction with which to demonstrate (e.g., \( \frac{1}{5} \)). Remind students that fractions represent quotients (e.g., \( \frac{1}{5} \) is the same as \( 5 \div 1 \)). So, to determine a percent from a fraction, divide the numerator by the denominator to get a quotient in hundredths that can be read as a percent (e.g., \( \frac{1}{5} = 5 \div 1 = 0.20 = \frac{20}{100} \), which is 20%). Do not have students multiply the decimal by 100 to get the percent; just have them convert mentally (i.e., \( 0.20 = \frac{20}{100} = 20\% \)), as multiplying by 100 will just add an extra step they will not need after Lesson 7, when they will learn the relationship between decimals and percents.

Reflecting

Here, students reflect on the relationships between fractions, ratios, and percents. They consolidate their understanding by communicating how they convert from one form to another, using examples. They realize that all are ways of expressing comparisons.
Answers

1. For example, the grid allowed James to visualize the ratio and the fraction as parts of 100.

2. For example, the fraction \( \frac{7}{20} \) and the ratio 4 : 25 are hard to compare because the whole is not the same. By writing an equivalent fraction and an equivalent ratio both with 100 as the whole, I could easily compare them and see that there were more occurrences of the finger being on the Pacific Ocean than on the Atlantic Ocean.

3. For example, the ratio 4 : 25 is equivalent to the ratio \( \frac{16}{100} \), which can be written as 16%. First, I write an equivalent fraction with 100 in the denominator, then I write the fraction as a percent.

4. a) \( 3 : 20 = 15 : 100 \)
   b) 15%

5. a) \( 12 : 100 = \frac{12}{100} = 12\% \)
   b) \( 91 : 100 = \frac{91}{100} = 91\% \)
   c) 0.01 = \( \frac{1}{100} = 1 : 100 = 1\% \)
   d) \( 50 : 100 = \frac{50}{100} = 50\% \)

6. a) \( 1 : 2 = 50 : 100 = 50\% \)
   b) \( 1 : 4 = 25 : 100 = 25\% \)

(Least 6 Answers continued on p. 86)

3. Consolidation • 20–30 min

Checking (Pairs)

For intervention strategies, refer to Meeting Individual Needs or the Assessment for Feedback chart.

4. Students may need to be prompted to review James’s Solution to help them answer Question 4.

Practising (Pairs/Individual)

Give students the choice of working in pairs or individually.

8. Ensure students understand that the “population” equals teachers plus students. Students may think they need to determine the actual population, but they don’t; what is asked for is ratios, which will show the proportions. Thus, in part a), for example, \( 4\% = \frac{4}{100} = 4 : 100; \) the actual population could be 50, 200, and so on. Ask students what information in their answers to parts a) and b) can be used to answer part c).

9. Here, in contrast to question 8, students are given the actual population of a class, which they can use to find a percent. They can compare this to the percent for the population of Canadian students without knowing that actual population.

Closing (Whole Class)

Have students summarize their learning about percent by having them explain how they could determine a percent in a real-life context. Stress that one determines percents by thinking about parts and wholes. What’s the part? What’s the whole? This gives a ratio (or fraction), which can then be expressed as a percent. Provide a sample situation for discussion. For example,

- Jane scored 6 points on a quiz with a possible total of 10. Express her grade as a percent.
**Assessment of Learning**—What to Look for in Student Work...

**Assessment Strategy: Written Answer**
Knowledge and Understanding

**Key Assessment Question 6**
- For each part to whole ratio, write an equivalent ratio using 100 as the whole. Use a hundredths grid. Write each ratio as a percent.
  - a) 1 to 2
  - b) 1 : 4
  - c) 6 to 25
  - d) 18 to 20

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- Demonstrates a **limited** or **inaccurate** understanding of the meaning of percent as a ratio with 100 as the whole
- Demonstrates some understanding of the meaning of percent as a ratio with 100 as the whole
- Demonstrates considerable understanding of the meaning of percent as a ratio with 100 as the whole
- Demonstrates thorough understanding of the meaning of percent as a ratio with 100 as the whole

**Extra Practice and Extension**
- You might assign any of the questions related to this lesson, which are cross-referenced in the chart below.

<table>
<thead>
<tr>
<th>Skills Bank</th>
<th>Problem Bank</th>
<th>Chapter Review</th>
<th>Workbook</th>
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</thead>
<tbody>
<tr>
<td>Student Book p. 380, Questions 11, 12, &amp; 13</td>
<td>Student Book p. 382, Question 5</td>
<td>Student Book p. 384, Questions 7 &amp; 8</td>
<td>p. 109, all questions</td>
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</tbody>
</table>

**Nelson Web Site**
Visit [mathK8.nelson.com](http://mathK8.nelson.com) and follow the links to *Nelson Mathematics 6*, Chapter 12.

**Math Background**
Percent is one of the most commonly used mathematical concepts. A percent is a part to whole ratio that compares a number or an amount to 100. Percents are a different way of recording concepts of fractions, decimals, and ratios. Initial instruction should build on familiar models, such as the 10-by-10 grid. Encourage students to use models or drawings to explain their work with percents.

**At Home**
- Students can locate examples of the use of percents in the media and express the percents as a ratio and a fraction.

**Optional: Chapter 12**
Mental Math p. 70